Name _____ Date _____

A Little Dash of Logic
Foundations for Proof

Vocabulary

Define each term in your own words.

1. induction

2. deduction

3. counterexample

4. conditional statement

5. propositional form

6. propositional variables

7. hypothesis

8. conclusion

9. truth value

10. truth table

Name _____ Date _____

6. Ava read an article that said eating too much sugar can lead to tooth decay and cavities. Ava noticed that her little brother Phillip eats a lot of sugar. She concludes that Phillip's teeth will decay and develop cavities.

Determine whether inductive reasoning or deductive reasoning is used in each situation. Then determine whether the conclusion is correct and explain your reasoning.

7. Jason sees a line of 10 school buses and notices that each is yellow. He concludes that all school buses must be yellow.

   It is inductive reasoning because he has observed specific examples of a phenomenon—the color of school buses—and come up with a general rule based on those specific examples.

   The conclusion is not necessarily true. It may be the case, for example, that all or most of the school buses in this school district are yellow, while another school district may have orange school buses.

8. Caitlyn has been told that every taxi in New York City is yellow. When she sees a red car in New York City, she concludes that it cannot be a taxi.

9. Miriam has been told that lightning never strikes twice in the same place. During a lightning storm, she sees a tree struck by lightning and goes to stand next to it, convinced that it is the safest place to be.

10. Jose is shown the first six numbers of a series of numbers: 7, 11, 15, 19, 23, 27. He concludes that the general rule for the series of numbers is $a_n = 4n + 3$.

2

11. Isabella sees 5 red fire trucks. She concludes that all fire trucks are red.

12. Carlos is told that all garter snakes are not venomous. He sees a garter snake in his backyard and concludes that it is not venomous.

In each situation, identify whether each person is using inductive or deductive reasoning. Then compare and contrast the two types of reasoning.

13. When Madison babysat for the Johnsons for the first time, she was there 2 hours and was paid $30. The next time she was there for 5 hours and was paid $75. She decided that the Johnsons were paying her $15 per hour. The third time she went, she stayed for 4 hours. She tells her friend Jennifer that she makes $15 per hour babysitting. So, Jennifer predicted that Madison made $60 for her 4-hour babysitting job.

Madison used inductive reasoning to conclude that the Johnsons were paying her at a rate of $15 per hour. From that general rule, Jennifer used deductive reasoning to conclude that 4 hours of babysitting should result in a payment of $60. The inductive reasoning looks at evidence and creates a general rule from the evidence. By contrast, the deductive reasoning starts with a general rule and makes a prediction or deduction about what will happen in a particular instance.

Name _____ Date _____

14. When Holly was young, the only birds she ever saw were black crows. So, she told her little brother Walter that all birds are black. When Walter saw a bluebird for the first time, he was sure it had to be something other than a bird.

15. Tamika is flipping a coin and recording the results. She records the following results: heads, tails, heads, tails, heads, tails, heads. She tells her friend Javon that the coin alternates between heads and tails for each toss. Javon tells her that the next time the coin is flipped, it will definitely be tails.

16. John likes to watch the long coal trains moving past his house. Over the weeks of watching he notices that every train going east is filled with coal, but the trains heading west are all empty. He tells his friend Richard that all trains heading east have coal and all trains heading west are empty. When Richard hears a train coming from the west, he concludes that it will certainly be filled with coal.

17. Vance earned $60 mowing 5 lawns last weekend for the Greenvalley Homeowners Association. Vance concluded that he earned $12 for each lawn. Vance told Sherwin that he planned to mow 7 lawns for Greenvalley next weekend. Sherwin concluded that Vance would earn $84 mowing the 7 lawns.

2

18. As a child, the only frogs Emily ever saw were green. Emily told Juan that all frogs are green. When Juan visited a zoo and saw a blue poison dart frog he concluded that it must be something other than a frog.

Write each statement in propositional form.

19. The measure of an angle is 90°. So, the angle is a right angle.

If the measure of an angle is 90°, then the angle is a right angle.

20. Three points are all located on the same line. So, the points are collinear points.

21. Two lines are not on the same plane. So, the lines are skew.

22. Two angles are supplementary angles if the sum of their angle measures is equal to 180°.

Name _____ Date _____

23. Two angles share a common vertex and a common side. So, the angles are adjacent angles.

24. A ray divides an angle into two congruent angles. So, the ray is an angle bisector.

Identify the hypothesis and the conclusion of each conditional statement.

25. If two lines intersect at right angles, then the lines are perpendicular.
The hypothesis is "Two lines intersect at right angles."
The conclusion is "The lines are perpendicular."

26. If the sum of two angles is 180°, then the angles are supplementary.

27. If the sum of two adjacent angles is 180°, then the angles form a linear pair.

28. If the measure of an angle is 180°, then the angle is a straight angle.

29. If two lines are located in the same plane, then the lines are coplanar lines.

30. If the sum of two angle measures is equal to 90°, then the angles are complementary angles.

Answer each question about the given conditional statement.

31. Conditional statement: If the measure of angle *ABC* is 45° and the measure of angle *XYZ* is 45°, then $\angle ABC = \angle XYZ$.

What does it mean if the hypothesis is false and the conclusion is true, and then what is the truth value of the conditional statement?

If the hypothesis is false and the conclusion is true, then the measure of angle *ABC* is not 45 degrees and the measure of angle *XYZ* is not 45 degrees, and angles *ABC* and *XYZ* are congruent. The truth value of the conditional statement is true, because the angles could have measures that are equal, but different than 45 degrees.

32. Conditional statement: If the measure of angle *XYZ* is less than 90°, then angle *XYZ* is acute.

What does it mean if the hypothesis is true and the conclusion is false, and then what is the truth value of the conditional statement?

33. Conditional statement: If $\angle 1$ and $\angle 2$ are two nonadjacent angles formed by two intersecting lines, then they are vertical angles.

What does it mean if the hypothesis is true and the conclusion is true, and then what is the truth value of the conditional statement?

34. Conditional statement: If the measure of $\angle LMN$ is 180°, then $\angle LMN$ is a straight angle.

What does it mean if the hypothesis is false and the conclusion is false, and then what is the truth value of the conditional statement?

Name _____ Date _____

For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

35. If $\overrightarrow{RT}$ bisects $\angle PRS$, then $\angle PRT$ and $\angle SRT$ are adjacent angles.

Given: $\overrightarrow{RT}$ bisects $\angle PRS$

Prove: $\angle PRT$ and $\angle SRT$ are adjacent angles

36. If $\angle QRS$ and $\angle SRT$ are complementary angles, then $m\angle QRS + m\angle SRT = 90°$.

Given:

Prove:

37. If $\overleftrightarrow{AB} \perp \overline{KJ}$ and $\overleftrightarrow{AB}$ bisects $\overline{KJ}$, then $\overleftrightarrow{AB}$ is the perpendicular bisector of $\overline{KJ}$.

Given:

Prove:

38. If $\overrightarrow{PG}$ bisects $\angle FPH$, then $\angle FPG \cong \angle GPH$.

2

Given:

Prove: