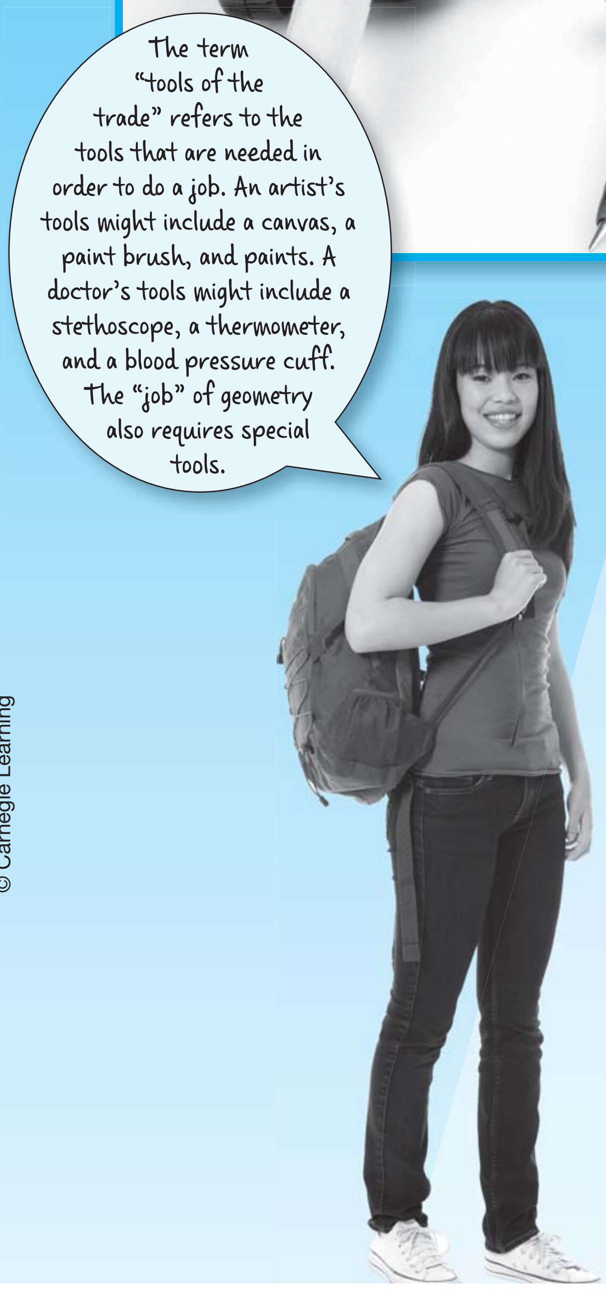
# Tools of Geometry

# 1


**1.1 Let's Get This Started!**
Points, Lines, Planes, Rays, and Line Segments . . . . . . . . 3

**1.2 Let's Move!**
Translating and Constructing Line Segments. . . . . . . . . . 17

**1.3 Treasure Hunt**
Midpoints and Bisectors. . . . . . . . . . . . . . . . . . . . . . . . . 35

**1.4 It's All About Angles**
Translating and Constructing Angles and Angle Bisectors. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 51

**1.5 Did You Find a Parking Space?**
Parallel and Perpendicular Lines on the Coordinate Plane . . . . . . . . . . . . . . . . . . . . . . . . . . . 61

**1.6 Making Copies—Just as Perfect as the Original!**
Constructing Perpendicular Lines, Parallel Lines, and Polygons . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 75

**1.7 What's the Point?**
Points of Concurrency . . . . . . . . . . . . . . . . . . . . . . . . . . . 87

# Let's Get This Started!

## Points, Lines, Planes, Rays, and Line Segments

1.1

### LEARNING GOALS

In this lesson, you will:

- Identify and name points, lines, planes, rays, and line segments.
- Use symbolic notation to describe points, lines, planes, rays, and line segments.
- Describe possible intersections of lines and planes.
- Identify construction tools.
- Distinguish between a sketch, a drawing, and a construction.

### KEY TERMS

- point
- line
- collinear points
- plane
- compass
- straightedge
- sketch
- draw
- construct
- coplanar lines
- skew lines
- ray
- endpoint of a ray
- line segment
- endpoints of a line segment
- congruent line segments

Do you have techno-joy or techno-fear? For people who have techno-fear, learning about a new technology—whether it's a smart phone, new computer, or new TV—can be a nervous endeavor. For people in the techno-joy category, it's, "get out of the way, I can handle this new device! I don't need to read the directions!"

Technical writers are the kind of people who bridge the gap between techno-joy and techno-fear. Technical writers are people who write and edit the manuals for all kinds of devices—from cars to airplanes, from electronic tablets to blenders, from industrial ventilation fans to electronic medical devices, from vacuums to whatever device you can think of! Most technical writers take the technological language and specifications and convert them into language that is more comprehensible to the average user. Some technical writers will also translate manuals into different languages.

Have you ever had trouble building something by reading the instructions? It's a tough job to write instructions, especially when you don't really know the audience.

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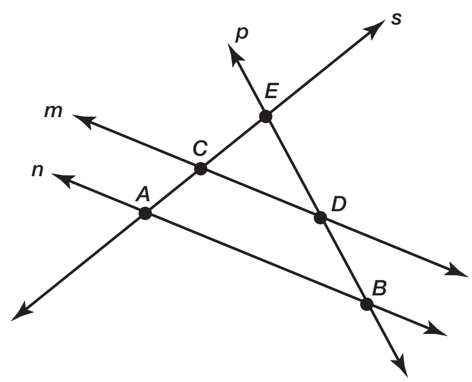
## PROBLEM 1 Point, Line, Plane

There are three essential building blocks of geometry—the point, the line, and the plane. These three terms are called undefined terms; we can only describe and create mathematical models to represent them.

A **point** is described simply as a location. A point in geometry has no size or shape, but it is often represented using a dot. In a diagram, a point can be labeled using a capital letter.

A **line** is described as a straight, continuous arrangement of an infinite number of points. A line has an infinite length, but no width. Arrowheads are used to indicate that a line extends infinitely in opposite directions. In a diagram, a line can be labeled with a lowercase letter positioned next to the arrowhead.

A mathematical model of several points and lines is shown.




1. Does the name "line *C*" describe a unique line? Explain why or why not.

2. Does the name "line *CD*" describe a unique line? Explain why or why not.

3. Does the name "line *m*" describe a unique line? Explain why or why not.

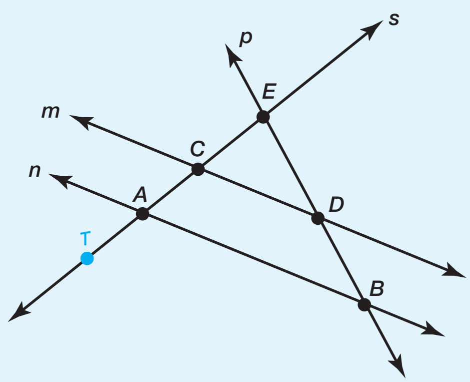

4. How many points are needed to name a specific line?

5. What is another name for line *AB*?

Line *AB* can be written using symbols as $\overleftrightarrow{AB}$ and is read as "line *AB*."

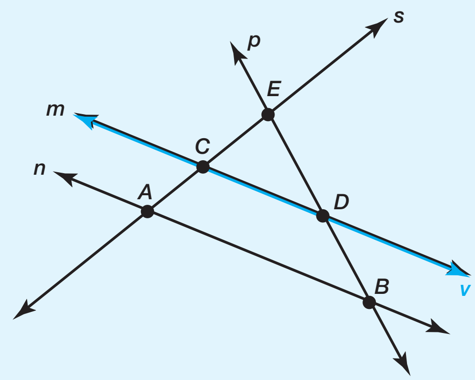
6. Analyze each model and explanation.

**Brad**




*I drew point T on line s. Since only one line goes through T, point T describes just one line.*

**Kara**




*I drew line v through points C and D. So, line CD describes two lines.*

Describe the inaccuracy in each students' reasoning.

7. How many lines can be drawn through a single point?

**Collinear points** are points that are located on the same line.

8. Use the diagram shown prior to Question 1.

   a. Name three points that are collinear.

   b. Name three points that are not collinear.

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A **plane** is described as a flat surface. A plane has an infinite length and width, but no depth, and extends infinitely in all directions. One real-world model of a plane is the surface of a still body of water. Three non-collinear points describe a unique plane, but planes are usually named using one italic letter located near a corner of the plane as drawn.

Three planes can intersect in a variety of ways or may not intersect at all.

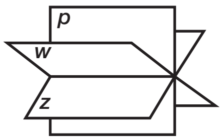

Figure 1

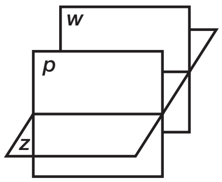

Figure 2

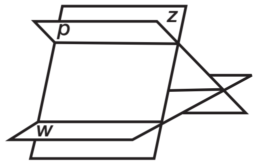

Figure 3

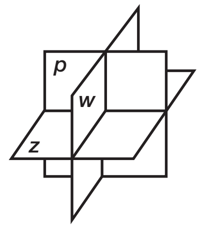

Figure 4

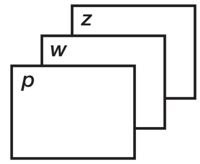

Figure 5

9. Describe the intersection of planes *p*, *w*, and *z* in each figure.

   **a.** Figure 1

   **b.** Figure 2

   **c.** Figure 3

   **d.** Figure 4

   **e.** Figure 5

10. List all of the possible ways that three planes can intersect.

11. Sketch and describe all possible ways that a line and a plane can intersect.

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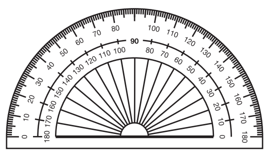
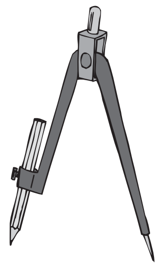
## PROBLEM 2 Creating Geometric Figures

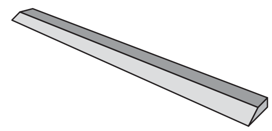
You can use many tools to create geometric figures. Some tools, such as a ruler or a protractor, are classified as measuring tools. A **compass** is a tool used to create arcs and circles. A **straightedge** is a ruler with no numbers. It is important to know when to use each tool.



Protractor



Compass



Straightedge

- When you **sketch** a geometric figure, the figure is created without the use of tools.
- When you **draw** a geometric figure, the figure is created with the use of tools such as a ruler, straightedge, compass, or protractor. A drawing is more accurate than a sketch.
- When you **construct** a geometric figure, the figure is created using only a compass and a straightedge.

**1.** Sketch and then draw each figure. Describe the steps that you performed to complete your sketch and your drawing.

**a.** square

**b.** isosceles triangle

**Coplanar lines** are two or more lines that are located in the same plane. **Skew lines** are two or more lines that do not intersect and are not parallel. Skew lines do not lie in the same plane.

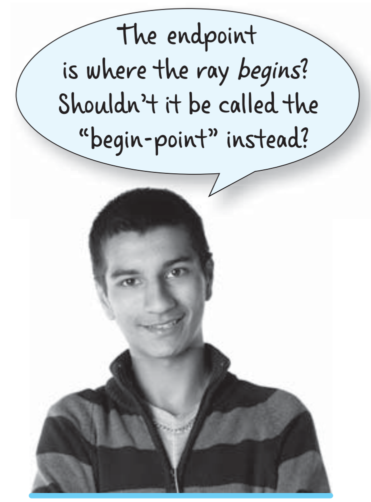
**2.** Draw and label three coplanar lines.

**3.** Look around your classroom. Describe the location of two skew lines.

## PROBLEM 3 Using Undefined Terms to Define New Terms

A **ray** is a part of a line that begins with a single point and extends infinitely in one direction. The **endpoint of a ray** is the single point where the ray begins.

A ray is named using two capital letters, the first representing the endpoint and the second representing any other point on the ray. Ray $AB$ can be written using symbols as $\overrightarrow{AB}$, which is read as "ray $AB$."

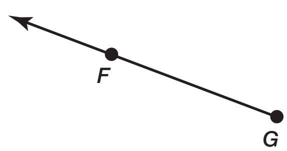



1. Sketch and label $\overrightarrow{AB}$.


2. Sketch and label $\overrightarrow{BA}$.


3. Are $\overrightarrow{AB}$ and $\overrightarrow{BA}$ names for the same ray? Explain why or why not.

4. Use symbols to name the geometric figure shown.



A **line segment** is a part of a line that includes two points and all of the collinear points between the two points. The **endpoints of a line segment** are the points where the line segment begins and ends.

A line segment is named using two capital letters representing the two endpoints of the line segment. Line segment *AB* can be written using symbols as $\overline{AB}$, which is read as "line segment *AB*."

**5.** Draw and label $\overline{AB}$.

*A* *B*

**6.** Draw and label $\overline{BA}$.

*A* *B*

**7.** Are $\overline{AB}$ and $\overline{BA}$ names for the same line segment? Explain why or why not.


**8.** Use a ruler to measure $\overline{AB}$ in Question 5.

**9.** The measure of $\overline{AB}$ can be expressed in two different ways. Complete each statement:

**a.** "$AB =$ ____________ inches" is read as "the distance from point *A* to point *B* is equal to ____________ inches."

**b.** "$m\overline{AB} =$ ____________ inches" is read as "the measure of line segment *AB* is equal to ____________ inches."

**c.** How do you read "$m\overline{CF} = 3$ inches"?

**d.** How do you read "$SP = 8$ inches"?

**10.** Use symbols to name each geometric figure.

**a.**


**b.**


**c.**

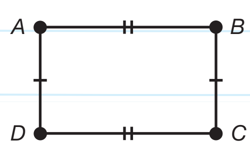

If two line segments have equal measure, then the line segments have the same length. **Congruent line segments** are two or more line segments of equal measure.

If $m\overline{AB} = m\overline{CD}$, then line segment *AB* is congruent to line segment *CD* by the definition of congruent line segments. This statement can be written using symbols as $\overline{AB} \cong \overline{CD}$ and is read as "line segment *AB* is congruent to line segment *CD*."

Use the congruence symbol, $\cong$, between references to congruent geometric figures; and the equal symbol, $=$, between references to equal lengths or distances.

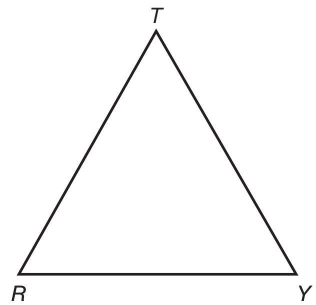
Markers are used to indicate congruent segments in geometric figures. If a diagram has more than one set of congruent segments then sets of markers can be used.

The figure shows $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$.




**11.** Draw and label two congruent line segments. Then, use symbols to write a statement that describes their relationship.

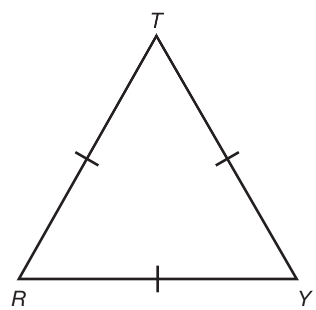
**12.** Ms. Snyder drew the triangle shown and asked her students to classify it.




**a.** Mariah says the triangle is an equilateral triangle.

Is she correct?

**b.** Ms. Snyder then drew markers and asked her students to classify the triangle.




Mariah says the triangle is an isosceles triangle. Justin says the triangle is an equilateral triangle.

Who is correct?

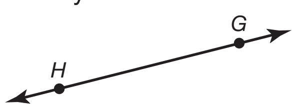
**c.** Ms. Snyder also asked her students to write a statement that best describes the congruency of the line segments forming the triangle.

| Mariah | Justin |
|---|---|
| $TR = RY = YR$ | $\overline{TR} \cong \overline{RY} \cong \overline{YR}$ |

Who is correct?

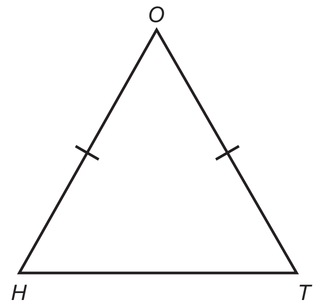
**13.** Use symbols to write three valid conclusions based on the figure shown. How do you read each conclusion?


**14.** Use symbols to name all lines, rays, or segments shown.




**15.** Explain when it is appropriate to use the statement $JK = MN$ and when it is appropriate to use the statement $\overline{JK} \cong \overline{MN}$.

## Talk the Talk




Circle each statement that is valid about triangle HOT.

| | |
|---|---|
| $\overline{HO} = \overline{TO}$ | $m\overline{HO} = m\overline{TO}$ |
| $\overline{HO} \cong \overline{TO}$ | $m\overline{HO} \cong m\overline{TO}$ |
| $HO = TO$ | $mHO = mTO$ |
| $HO \cong TO$ | $mHO \cong mTO$ |



Be prepared to share your solutions and methods.

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# 1.2

# Let's Move!

## Translating and Constructing Line Segments

### LEARNING GOALS

In this lesson, you will:

- Determine the distance between two points.
- Use the Pythagorean Theorem to derive the Distance Formula.
- Apply the Distance Formula on the coordinate plane.
- Translate a line segment on the coordinate plane.
- Copy or duplicate a line segment by construction.

### KEY TERMS

- Distance Formula
- transformation
- rigid motion
- translation
- pre-image
- image
- arc

### CONSTRUCTIONS

- copying a line segment
- duplicating a line segment

Are you better at geometry or algebra? Many students have a preference for one subject or the other; however, geometry and algebra are very closely related. While there are some branches of geometry that do not use much algebra, analytic geometry applies methods of algebra to geometric questions. Analytic geometry is also known as the study of geometry using a coordinate system. So anytime you are performing geometric calculations and it involves a coordinate system, you are studying analytic geometry. Be sure to thank Descartes and his discovery of the coordinate plane for this!

What might be the pros and cons of analytic geometry compared to other branches of geometry? Does knowing about analytic geometry change how you feel about your own abilities in geometry or algebra?

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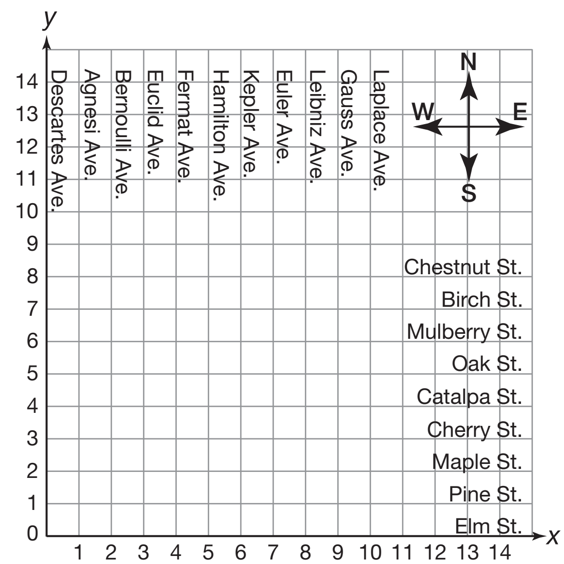
## PROBLEM 1 Where Do You Live?

Don, Freda, and Bert live in a town where the streets are laid out in a grid system.

1. Don lives 3 blocks east of Descartes Avenue and 5 blocks north of Elm Street. Freda lives 7 blocks east of Descartes Avenue and 2 blocks north of Elm Street. Plot points to show the locations of Don's house and Freda's house on the coordinate plane. Label each location with the student's name and the coordinates of the point.




a. Name the intersection of streets that Don lives on.

b. Name the intersection of streets that Freda lives on.

2. Bert lives at the intersection of the avenue that Don lives on, and the street that Freda lives on. Plot and label the location of Bert's house on the coordinate plane. Describe the location of Bert's house with respect to Descartes Avenue and Elm Street.

3. How do the *x*- and *y*-coordinates of Bert's house compare to the *x*- and *y*-coordinates of Don's house and Freda's house?

4. Use Don's and Bert's house coordinates to write and simplify an expression that represents the distance between their houses. Explain what this means in terms of the problem situation.

5. Use Bert's and Freda's house coordinates to write and simplify an expression that represents the distance between their houses. Explain what this means in terms of the problem situation.

6. All three friends are planning to meet at Don's house to hang out. Freda walks to Bert's house, and then Freda and Bert walk together to Don's house.

   a. Use the coordinates to write and simplify an expression that represents the total distance from Freda's house to Bert's house to Don's house.

   b. How far, in blocks, does Freda walk altogether?

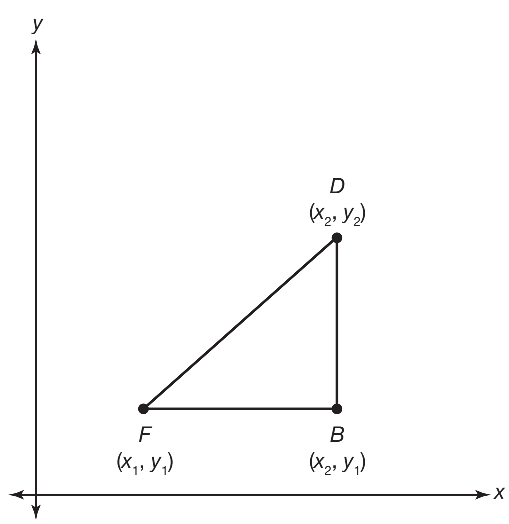
7. Draw the direct path from Don's house to Freda's house on the coordinate plane. If Freda walks to Don's house on this path, how far, in blocks, does she walk? Explain how you determined your answer.

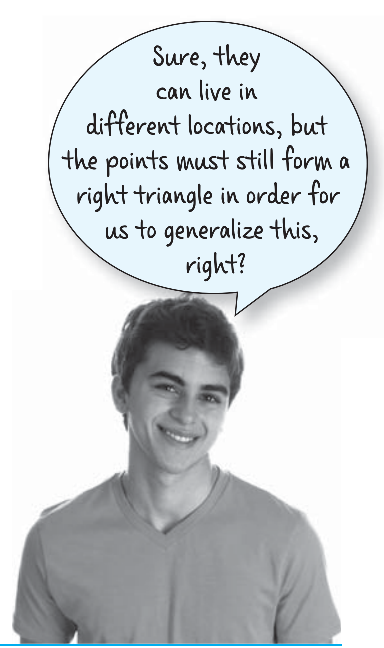
1

**8.** Complete the summary of the steps that you took to determine the direct distance between Freda's house and Don's house. Let *d* be the direct distance between Don's house and Freda's house.

| Distance between Bert's house and Freda's house | | Distance between Don's house and Bert's house | | Direct distance between Don's house and Freda's house |
|---|---|---|---|---|
| $(\square - \square)^2$ | + | $(\square - \square)^2$ | = | $\square$ |
| $\square^2$ | + | $\square^2$ | = | $\square$ |
| $\square$ | + | $\square$ | = | $\square$ |
| | | | $\square$ = | $\square$ |
| | | | $\square$ = | $\square$ |

Suppose Freda's, Bert's, and Don's houses were at different locations but oriented in a similar manner. You can generalize their locations by using $x_1$, $x_2$, $y_1$, and $y_2$ and still solve for the distances between their houses using variables. Let point *F* represent Freda's house, point *B* represent Bert's house, and point *D* represent Don's house.




**9.** Use the graph to write an expression for each distance.

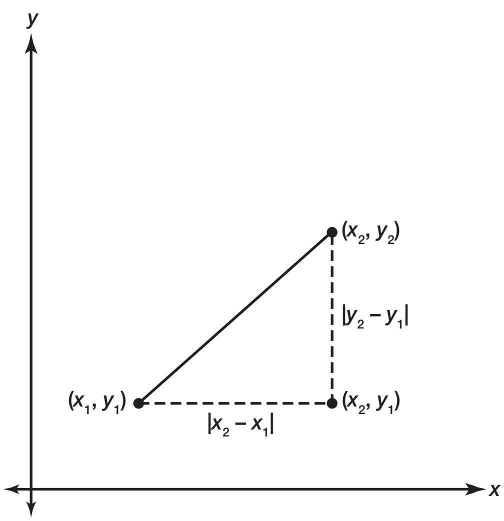
**a.** Don's house to Bert's house (*DB*)

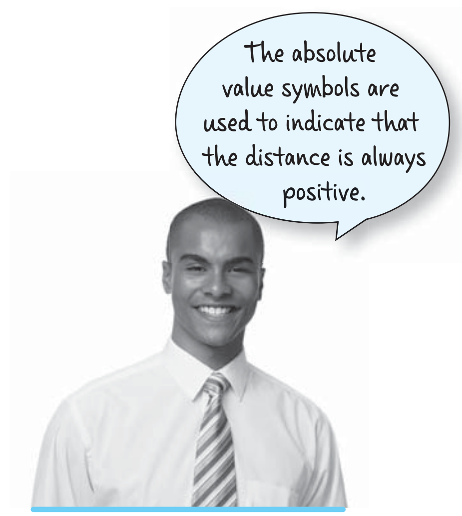
**b.** Bert's house to Freda's house (*BF*)

**10.** Use the Pythagorean Theorem to determine the distance from Don's house to Freda's house ($DF$).

You used the Pythagorean Theorem to calculate the distance between two points on the coordinate plane. Your method can be written as the *Distance Formula.*

The **Distance Formula** states that if $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the coordinate plane, then the distance $d$ between $(x_1, y_1)$ and $(x_2, y_2)$ is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.




**11.** Do you think that it matters which point you identify as $(x_1, y_1)$ and which point you identify as $(x_2, y_2)$ when you use the Distance Formula? Use an example to justify your answer.

12. Calculate the distance between each pair of points. Round your answer to the nearest tenth if necessary. Show all your work.

a. (1, 2) and (3, 7)

b. (−6, 4) and (2, −8)

c. (−5, 2) and (−6, 10)

d. (−1, −3) and (−5, −2)

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**13.** Carlos and Mandy just completed Question 12 parts (a) through (c). Now, they need to calculate the distance between the points (−4, 2) and (−2, 7). They notice the similarity between this problem and part (a).

Mandy

$d = \sqrt{(-4 - -2)^2 + (2 - 7)^2}$

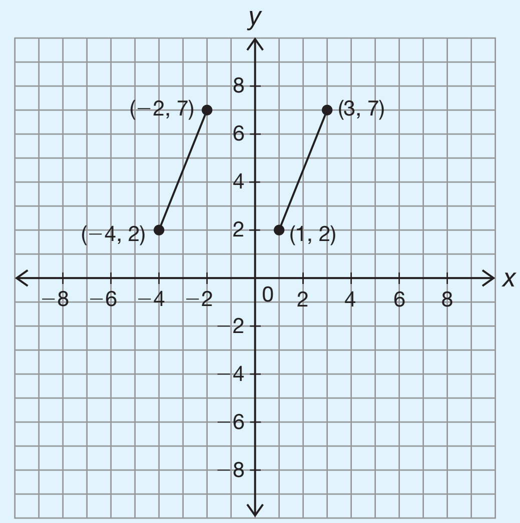
$d = \sqrt{(-2)^2 + (-5)^2}$

$d = \sqrt{4 + 25}$

$d = \sqrt{29}$

$d \approx 5.4$

Carlos




$(1, 2) \rightarrow (-4, 2)$ The point moved 5 units to the left

$(3, 7) \rightarrow (-2, 7)$ The point moved 5 units to the left

Since both points moved 5 units to the left, this did not alter the distance between the points, so the distance between points $(-4, 2)$ and $(-2, 7)$ is approximately 5.4.

Who used correct reasoning?

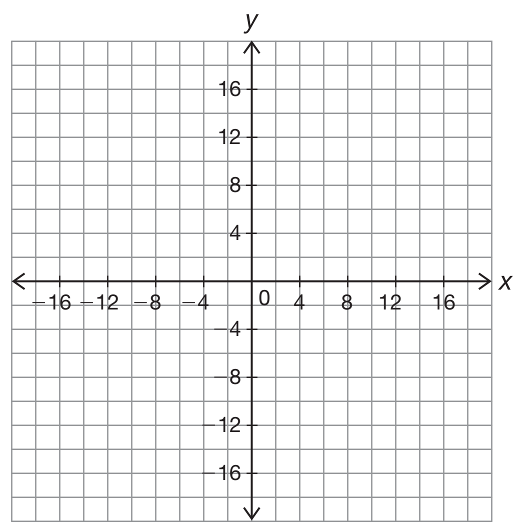
**14.** The distance between (*x*, 2) and (0, 6) is 5 units. Use the Distance Formula to determine the value of *x*. Show all your work.

## PROBLEM 2 Translating a Line Segment

1. Pedro's house is located at (6, 10). Graph this location on the coordinate plane and label the point *P*.

2. Jethro's house is located at (2, 3). Graph this location on the coordinate plane and label the point *J*.

3. Draw a line segment connecting the two houses to create line segment *PJ*.



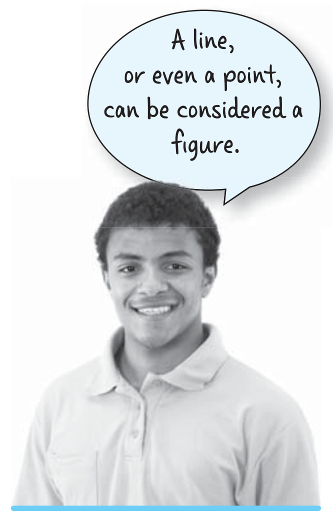

4. Determine the length of line segment *PJ*.

A **transformation** is the mapping, or movement, of all the points of a figure in a plane according to a common operation.

A **rigid motion** is a transformation of points in space.

A **translation** is a rigid motion that "slides" each point of a figure the same distance and direction. Sliding a figure left or right is a horizontal translation, and sliding it up or down is a vertical translation.

The original figure is called the **pre-image**. The new figure created from the translation is called the **image**.




**5.** Line segment $PJ$ is horizontally translated 10 units to the left.

**a.** Graph the image of pre-image $\overline{PJ}$. Label the new points $P'$ and $J'$.

**b.** Identify the coordinates of $P'$ and $J'$.

**6.** Line segment $P'J'$ is vertically translated 14 units down.

**a.** Graph the image of pre-image $\overline{P'J'}$. Label the new points $P''$ and $J''$.


**b.** Identify the coordinates of $P''$ and $J''$.

**7.** Line segment $P''J''$ is horizontally translated 10 units to the right.

**a.** Without graphing, predict the coordinates of $P'''$ and $J'''$.

**b.** Graph the image of pre-image $\overline{P''J''}$. Label the new points $P'''$ and $J'''$.

8. Describe the translation necessary on $\overline{P'''J'''}$ so that it returns to the location of $\overline{PJ}$.

9. How do the lengths of the images compare to the lengths of the pre-images? Explain how you could verify your answer.

10. Analyze the coordinates of the endpoints of each line segment.

a. Identify the coordinates of each line segment in the table.

| **Line Segments** | $\overline{PJ}$ | $\overline{P'J'}$ | $\overline{P''J''}$ | $\overline{P'''J'''}$ |
|---|---|---|---|---|
| **Coordinates of Endpoints** | | | | |

b. Describe how a horizontal translation changes the *x*- and *y*-coordinates of the endpoints.

c. Describe how a vertical translation changes the *x*- and *y*-coordinates of the endpoints.

11. Describe a sequence of two translations that will result in the image and the pre-image being the same.

12. Describe a sequence of four translations that will result in the image and the pre-image being the same.

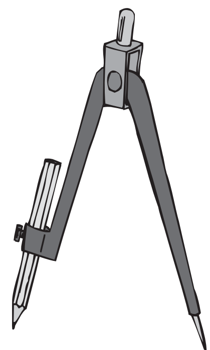
## PROBLEM 3 Copying Line Segments

In the previous problem, you translated line segments on the coordinate plane. The lengths of the line segments on the coordinate plane are measurable.

In this problem, you will translate line segments when measuring is not possible. This basic geometric construction is called **copying a line segment** or **duplicating a line segment.** You will perform the construction using a compass and a straightedge.

One method for copying a line segment is to use circles. But before you can get to that, let's review how to draw perfect circles with a compass.

Remember that a compass is an instrument used to draw circles and arcs. A compass can have two legs connected at one end.

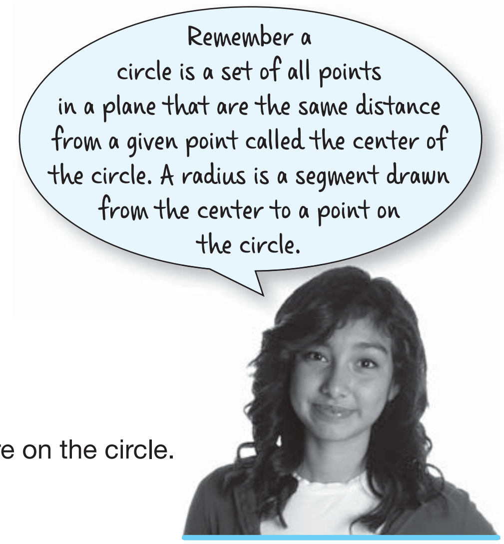


One leg has a point, and the other holds a pencil. Some newer compasses may be different, but all of them are made to construct circles by placing a point firmly into the paper and then spinning the top of the compass around, with the pencil point just touching the paper.

1. Use your compass to construct a number of circles of different sizes.

2. Point $C$ is the center of a circle and $\overline{CD}$ is a radius.

   a. Construct circle $C$.


   b. Draw and label points $A$, $B$, $E$, and $F$ anywhere on the circle.

   c. Construct $\overline{AC}$, $\overline{BC}$, $\overline{EC}$, and $\overline{FC}$.

   d. Shawna makes the following statement about radii of a circle.

> **Shawna**
>
> *All radii are the same length, because all of the points of a circle are equidistant from the circle's center.*

   Explain how Shawna knows that all radii are the same length? Does this mean the line segments you constructed are also radii?

An **arc** is a part of a circle. You can also think of an arc as the curve between two points on the circle.

3. Point $C$ is the center of a circle and $\overline{AC}$ is the radius.

   a. Construct an arc of circle $C$. Make your arc about one-half inch long. Construct the arc so that it does not pass through point $A$.


   b. Draw and label two points $B$ and $E$ on the arc and construct $\overline{CE}$ and $\overline{CB}$.

   c. What conclusion can you make about the constructed line segments?

Recall that congruent line segments are line segments that have the same length. The radii of a circle are congruent line segments because any line segment drawn from the center to a point on the circle has the same length.

**4.** Construct a circle with the center *A* and a radius of about 1 inch.

**a.** Without changing the width of your compass, place the compass point on any point on the circle you constructed and then construct another circle.

**b.** Draw a dot on a point where the two circles intersect. Place the compass point on that point of intersection of the two circles, and then construct another circle.

**c.** Repeat this process until no new circles can be constructed.

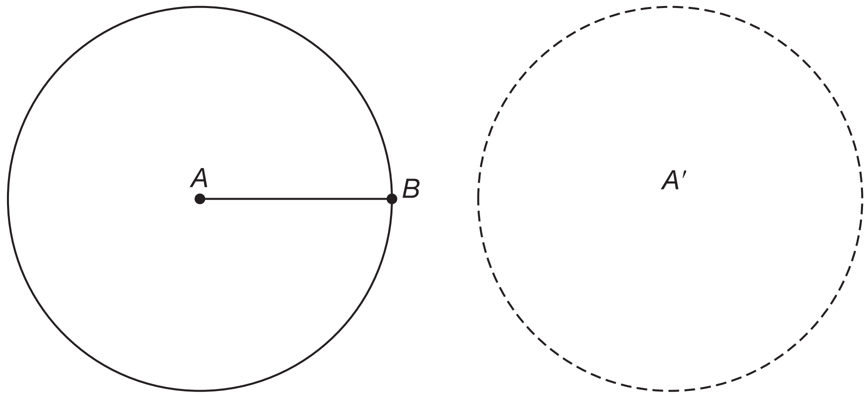
**d.** Connect the points of the circles' intersections with each other.




**e.** Describe the figure formed by the line segments.

Now let's use these circle-drawing skills to duplicate a line segment.

**5.** Circle $A$ is congruent to Circle $A'$.




**a.** Duplicate $\overline{AB}$ in Circle $A'$. Use point $A'$ as the center of the circle, then label the endpoint of the duplicated segment as point $B'$.

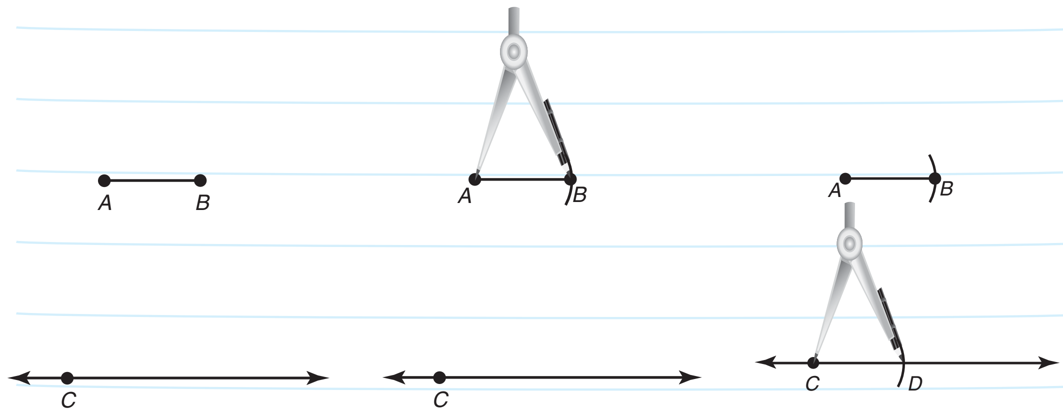
**b.** Describe the location of point $B'$.

**c.** If possible, construct a second line segment in Circle $A'$ that is a duplicate of $\overline{AB}$. Label the duplicate segment $\overline{A'C'}$. If it is not possible, explain why.

## PROBLEM 4 Another Method

To duplicate a line segment, you don't have to draw a full circle.

You can duplicate a line segment by constructing an exact copy of the original line segment.




**Construct a Starter Line**

Use a straightedge to construct a starter line longer than $\overline{AB}$. Label point *C* on the line.

**Measure Length**

Set your compass at the length *AB*.

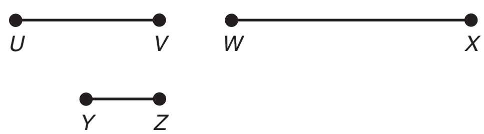
**Copy Length**

Place the compass at *C*. Mark point *D* on the new segment.

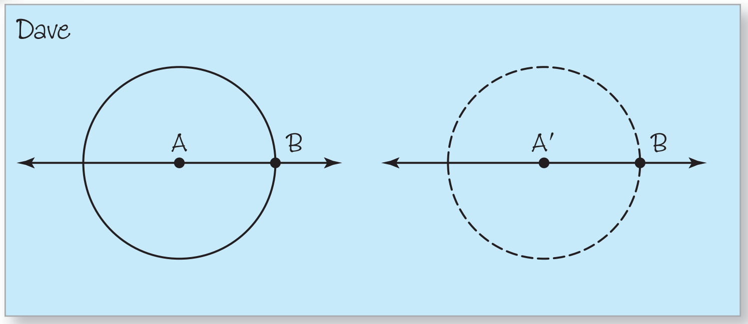
Line segment *CD* is a duplicate of line segment *AB*.

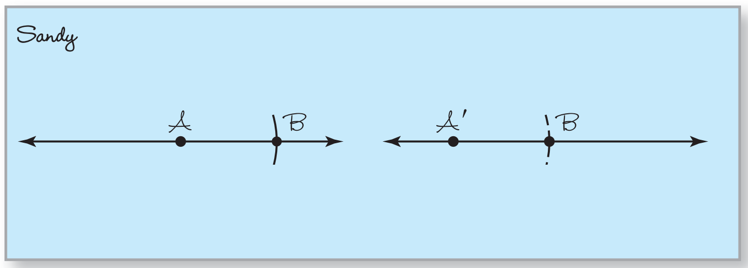
1. Construct a line segment that is twice the length of $\overline{AB}$.


2. Duplicate each line segment using a compass and a straightedge.



3. Dave and Sandy are duplicating $\overline{AB}$. Their methods are shown.




Which method is correct? Explain your reasoning.

4. Which method do you prefer? Why?

1

## Talk the Talk

1. You translate a line segment vertically or horizontally. Is the length of the image the same as the length of the pre-image? Explain why or why not.

2. If both endpoints of a line segment were not moved the same distance or direction, would the length of the line segment change? Would this still be considered a translation? Explain your reasoning.

3. What can you conclude about the length of a line segment and the length of its translated image that results from moving points on a coordinate plane the same distance and the same direction?

4. What can you conclude about the length of a line segment and the length of its translated image that results from construction?



Be prepared to share your solutions and methods.

# 1.3

# Treasure Hunt

## Midpoints and Bisectors

### LEARNING GOALS

In this lesson, you will:

- Determine the midpoint of a line segment on a coordinate plane.
- Use the Midpoint Formula.
- Apply the Midpoint Formula on the coordinate plane.
- Bisect a line segment using patty paper.
- Bisect a line segment by construction.
- Locate the midpoint of a line segment.

### KEY TERMS

- midpoint
- Midpoint Formula
- segment bisector

### CONSTRUCTIONS

- bisecting a line segment

When you hear the phrase "treasure hunt," you may think of pirates, buried treasure, and treasure maps. However, there are very few documented cases of pirates actually burying treasure, and there are no historical pirate treasure maps! So where did this idea come from?

Robert Louis Stevenson's book *Treasure Island* is a story all about pirates and their buried gold, and this book greatly influenced public knowledge of pirates. In fact, it is Stevenson who is often credited with coming up with the concept of the treasure map and using an X to mark where a treasure is located.

Have you ever used a map to determine your location or the location of another object? Did you find it difficult or easy to use? How does the idea of a treasure map relate to a familiar mathematical concept you are very familiar with?

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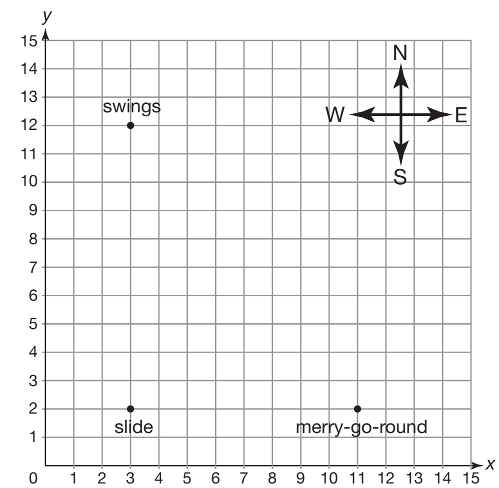
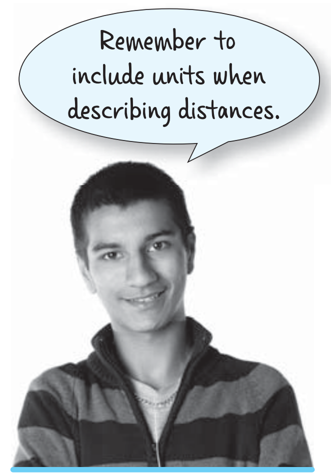
## PROBLEM 1 Locating the Treasure

Ms. Lopez is planning a treasure hunt for her kindergarten students. She drew a model of the playground on a coordinate plane as shown. She used this model to decide where to place items for the treasure hunt, and to determine how to write the treasure hunt instructions. Each grid square represents one square yard on the playground.




1. Ms. Lopez wants to place some beads in the grass halfway between the merry-go-round and the slide.

   a. Determine the distance between the merry-go-round and the slide. Show all your work.

   b. How far should the beads be placed from the merry-go-round and the slide?

   c. Write the coordinates for the location exactly halfway between the merry-go-round and the slide. Graph a point representing the location of the beads on the coordinate plane.

   d. How do the *x*- and *y*-coordinates of the point representing the location of the beads compare to the coordinates of the points representing the locations of the slide and the merry-go-round?

**2.** Ms. Lopez wants to place some kazoos in the grass halfway between the slide and the swings.

**a.** Write the coordinates for the location of the kazoos. Graph the location of the kazoos on the coordinate plane.

**b.** How do the *x*- and *y*-coordinates of the point representing the location of the kazoos compare to the coordinates of the points representing the locations of the slide and the swings?

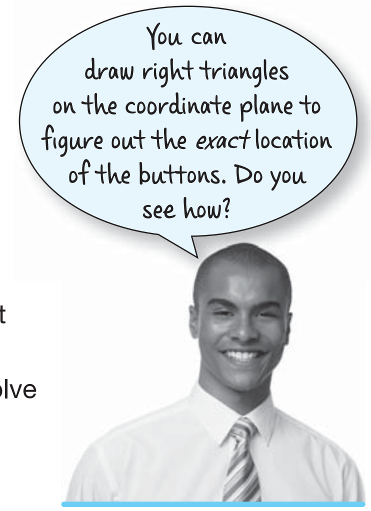
**3.** Ms. Lopez wants to place some buttons in the grass halfway between the swings and the merry-go-round.

**a.** Determine the distance between the swings and the merry-go-round.

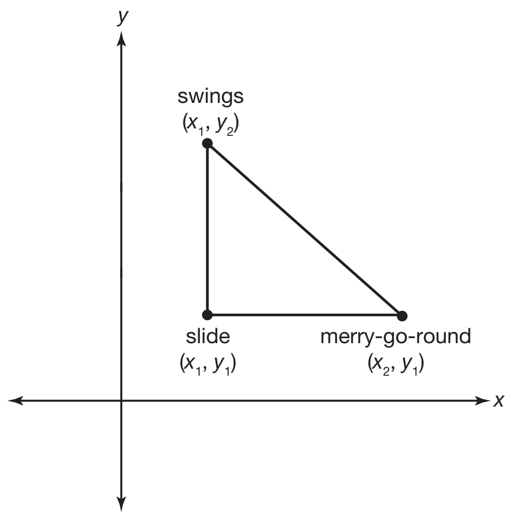
**b.** How far should the buttons be placed from the swings and the merry-go-round?

**c.** How is determining the coordinates for the location of the buttons different than determining the coordinates for the locations of the beads or the kazoos?

**d.** Write the coordinates for the location of the kazoos. Graph the location of the buttons on the coordinate plane.




Suppose the slide, the swings, and the merry-go-round were at different locations but oriented in a similar manner. You can generalize their locations by using $x_1$, $x_2$, $y_1$, and $y_2$, and then solve for the distances between each using variables.




**4.** Use the diagram to describe each distance algebraically.

**a.** the vertical distance from the *x*-axis to the slide

**b.** the distance from the slide to the swings

**c.** half the distance from the slide to the swings

**d.** the vertical distance from the *x*-axis to the slide plus half the distance from the slide to the swings

**5.** Simplify your expression from Question 4, part (d).

**6.** Use the diagram to describe each distance algebraically.

**a.** the horizontal distance from the *y*-axis to the slide

**b.** the distance from the slide to the merry-go-round

**c.** half the distance from the slide to the merry-go-round

**d.** the horizontal distance from the *y*-axis to the slide plus half the distance from the slide to the merry-go-round

**7.** Simplify your expression from Question 6, part (d).

The coordinates of the points that you determined in Questions 5 and 7 are *midpoints*. A **midpoint** is a point that is exactly halfway between two given points. The calculations you performed can be summarized by the *Midpoint Formula*.

The **Midpoint Formula** states that if $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

**8.** Use the Midpoint Formula to determine the location of the buttons from Question 3.

9. Do you think it matters which point you identify as $(x_1, y_1)$ and which point you identify as $(x_2, y_2)$ when you use the Midpoint Formula? Explain why or why not.

10. Determine the midpoint of each line segment from the given endpoints. Show all of your work.

**a.** (0, 5) and (4, 3)

**b.** (8, 2) and (6, 0)

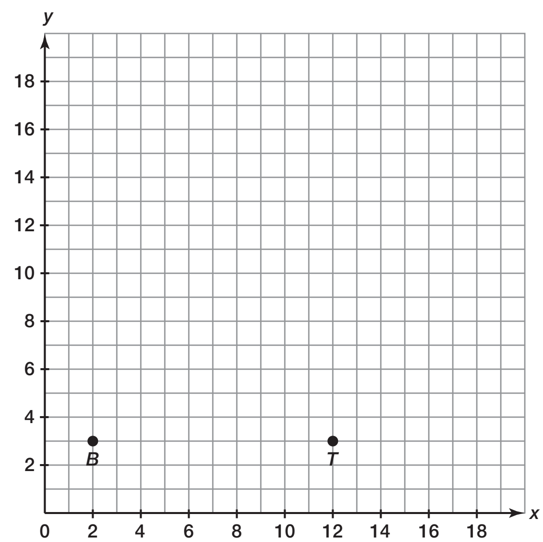
**c.** (−3, 1) and (9, −7)

**d.** (−10, 7) and (−4, −7)

## PROBLEM 2 Jack's Spare Key

1. Jack buried a spare key to his house in the backyard in case of an emergency. He remembers burying the key halfway between the back door and an oak tree. The location of the back door is point $B(2, 3)$, and the location of the oak tree is point $T(12, 3)$.

   a. Determine the location of the key. Show all of your work. Then graph the location of the key as point $M$.




   b. Suppose Jack buried his spare key $\frac{1}{3}$ of the way between the back door and the oak tree. Determine the location of the key. Show all of your work.

2. Jean also buried her house key. She remembers burying the key between the front porch and a rose bush. The location of the front porch is point $P(1, 2)$ and the location of the rose bush is point $B(16, 14)$.

   a. Suppose Jean buried her key $\frac{1}{2}$ of the way between the front porch and the rose bush. Determine the location of the key. Show all your work.

   b. Suppose Jean buried her key $\frac{1}{4}$ of the way between the front porch and the rose bush. Determine the location of the key. Show all your work.

   c. Suppose Jean buried her key $\frac{1}{3}$ of the way between the front porch and the rose bush. Explain why the Midpoint Formula is not helpful in determining the location of Jean's spare key.

**3.** Rick and Courtney used different methods to determine the location that is $\frac{1}{3}$ of the way between the front porch and the rose bush.

**Rick**

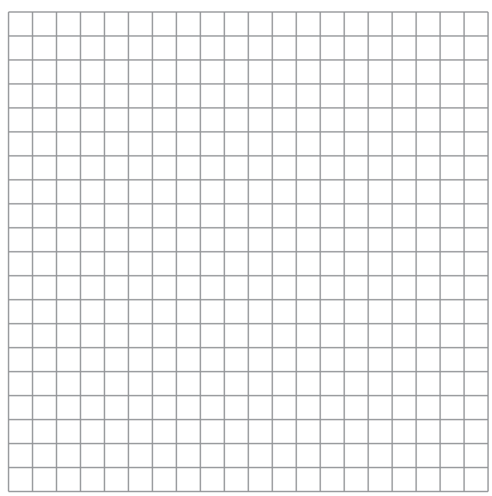
*I drew a vertical line from point B and a horizontal line from point P and determined the coordinates of point Z where they intersected. I divided the horizontal line and the vertical line into 3 equal parts and wrote the coordinates. I used those coordinates to divide segment BP into three parts. Then, I calculated the coordinates along BP.*

**Courtney**

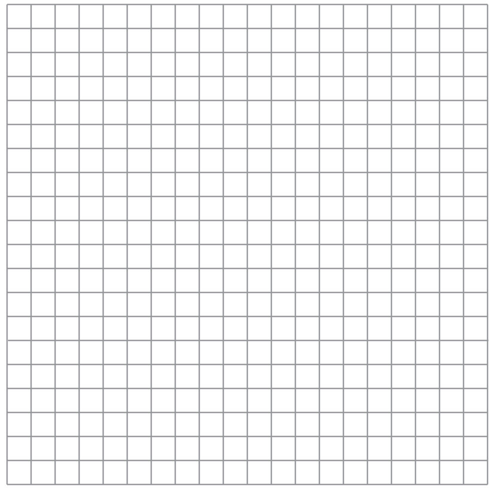
*I thought about the slope of segment BP. If I want to divide segment BP into three parts then I need to start at point P and perform three vertical and horizontal shifts to get to point B. So I can just divide the total horizontal and vertical shifts by three to calculate each vertical and horizontal shift. After that, determining the coordinates is a snap!*

Calculate the location of the point that is $\frac{1}{3}$ of the way between the front porch and the rose bush using Rick's method and Courtney's method. Use a graph to support your work.

**a.** Rick's Method



**b.** Courtney's Method



**4.** Suppose Jean buried her key $\frac{1}{5}$ of the way between the front porch and the rose bush. Determine the location of the key. Show all your work.

## PROBLEM 3 Stuck in the Middle

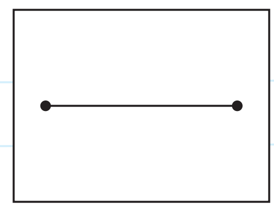
In the previous problem, you located the midpoint of a line segment on the coordinate plane. The lengths of the line segments on the plane are measurable.

In this problem, you will locate the midpoint of a line segment when measurement is not possible. This basic geometric construction used to locate a midpoint of a line segment is called **bisecting a line segment**. When bisecting a line segment, you create a *segment bisector*. A **segment bisector** is a line, line segment, or ray that divides a line segment into two line segments of equal measure, or two congruent line segments.

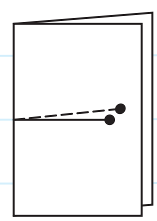
Just as with duplicating a line segment, there are a number of methods to bisect a line segment.

You can use tracing paper—also known as patty paper—to bisect a line.

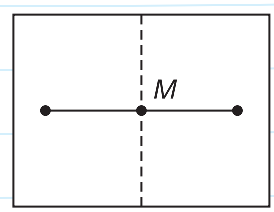
Draw a line segment on the paper.



Fold the paper so that the endpoints of the line segment lie on top of each other.



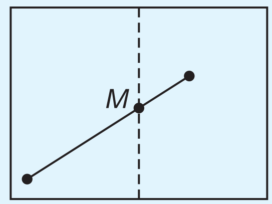
Open the paper. The crease represents the segment bisector, and the midpoint is located where the crease intersects the line segment.




1. Use tracing paper to duplicate a line segment. How do you know your bisector and midpoint are accurate?

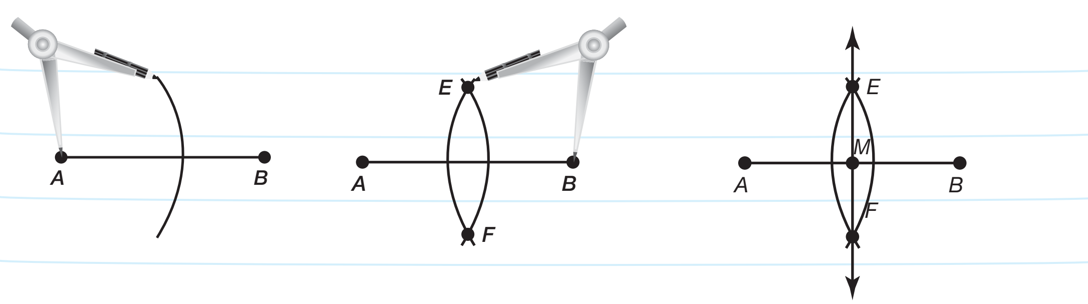
2. Thomas determined the midpoint of $\overline{AB}$ incorrectly.

**Thomas**




Explain what Thomas did incorrectly and how you can tell he is incorrect. Explain how he can correctly determine the midpoint of $\overline{AB}$.

You can use a compass and straightedge to construct a segment bisector.




**Construct an Arc**

Open the radius of the compass to more than half the length of line segment *AB*. Use endpoint *A* as the center and construct an arc.

**Construct Another Arc**

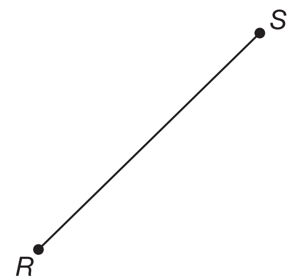
Keep the compass radius and use point *B* as the center as you construct an arc. Label the points formed by the intersection of the arcs point *E* and point *F*.

**Construct a Line**

Connect points *E* and *F*. Line segment *EF* is the segment bisector of line segment *AB*. The point *M* represents the midpoint of $\overline{AB}$.

Line *EF* bisects line segment *AB*. Point *M* is the midpoint of line segment *AB*.

**3.** Aaron is determining the midpoint of line segment *RS*. His work is shown.



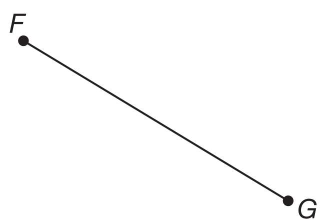

He states that because the arcs do not intersect, this line segment does not have a midpoint. Kate disagrees and tells him he drew his arcs incorrectly and that he must redraw his arcs to determine the midpoint. Who is correct? Explain your reasoning.

**4.** Use construction tools to locate the midpoint of each given line segment. Label each midpoint as *M*.

**a.**

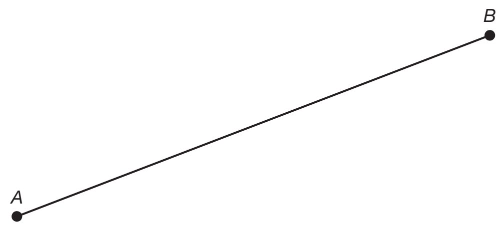
**b.**


**c.**



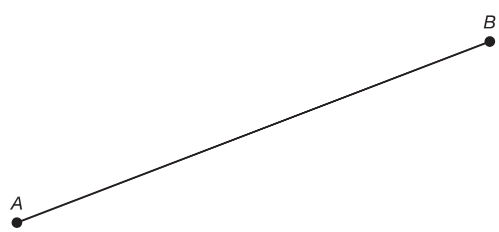
5. Perform each construction shown. Then explain how you performed each construction.

   a. Locate a point one-fourth the distance between point $A$ and point $B$.





   b. Locate a point one-third the distance between point $A$ and point $B$.




6. Explain how you can duplicate a line segment to verify that the midpoint resulting from bisecting the line segment is truly the midpoint of the segment.

1

## Talk the Talk

1. When bisecting a line segment using construction tools, does it make a difference which endpoint you use to draw the first arc?

2. When locating the midpoint of a line segment on a coordinate plane using the Midpoint Formula, does it make a difference which endpoint you use as $x_1$ and $y_1$?

3. How will you decide if you should use the Midpoint Formula or construction tools to locate a midpoint?



Be prepared to share your solutions and methods.

# 1.4

# It's All About Angles

## Translating and Constructing Angles and Angle Bisectors

**LEARNING GOALS**

In this lesson, you will:

- Translate an angle on the coordinate plane.
- Copy or duplicate an angle by construction.
- Bisect an angle by construction.

**KEY TERMS**

- angle
- angle bisector

**CONSTRUCTIONS**

- copying an angle
- duplicating an angle
- bisecting an angle

You may have never thought of it this way, but drawing and geometry are closely linked. Drawing is the process of deliberately arranging lines and curves to create an image. Most drawings have a number of different angles that are created through the intersection of these lines and curves. However, an art movement known as De Stijl (pronounced duh SHTEEL) limits drawings to using only horizontal and vertical lines. They also limit the colors used to the primary colors. While you may think this sounds restricting, many artists have created many works of art in this style. In fact, an architect even designed a house adhering to the De Stijl principles!

If De Stijl limits the artists to only using horizontal and vertical lines, what types of angles can be created in their art work? What types of angles cannot be created? What might be some challenges with drawing or painting in this style?

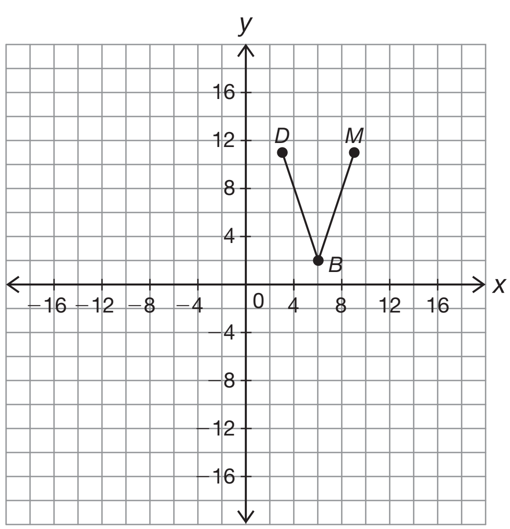
## PROBLEM 1 Translating an Angle

Previously, you practiced translating a line segment on the coordinate plane horizontally or vertically.

1. Describe how to translate a line segment on a coordinate plane.

An **angle** is formed by two rays or line segments that share a common endpoint. The sides of the angle are represented by the two rays or segments. Each ray of an angle contains an infinite number of line segments. The ∠ symbol represents "angle." Angle *DMB* can be written as ∠*DMB*.


2. Analyze ∠*DBM* shown on the coordinate plane. Describe how you would translate this angle on the coordinate plane.



3. Complete each translation.

   a. Horizontally translate $\angle DBM$ 13 units left. Label the image $\angle D'B'M'$.

   b. Vertically translate $\angle D'B'M'$ 15 units down. Label the image $\angle D''B''M''$.

   c. Horizontally translate $\angle D''B''M''$ 13 units right. Label the image $\angle D'''B'''M'''$.

   d. Use the graph to complete the tables by determining the endpoints of each line segment.

| **Line Segments** | $\overline{MB}$ | $\overline{M'B'}$ | $\overline{M''B''}$ | $\overline{M'''B'''}$ |
|---|---|---|---|---|
| **Coordinates of Endpoints** | | | | |

| **Line Segments** | $\overline{DB}$ | $\overline{D'B'}$ | $\overline{D''B''}$ | $\overline{D'''B'''}$ |
|---|---|---|---|---|
| **Coordinates of Endpoints** | | | | |

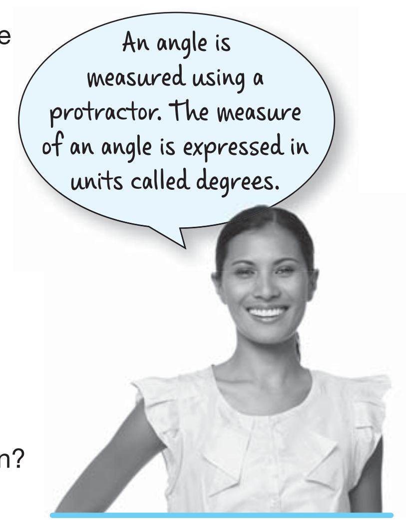
4. Describe how a horizontal translation changes the *x*- and *y*-coordinates of the endpoints of each side of an angle.

5. Describe how a vertical translation changes the *x*- and *y*-coordinates of the angle endpoints of each side of an angle.

6. Describe a sequence of two translations that will result in the image and the pre-image of an angle being the same.

7. Describe a sequence of four translations that will result in the image and the pre-image of an angle being the same.

8. Measure each angle on the coordinate plane. How do the measures of each image compare to its corresponding pre-image?




9. What is the result of moving only one angle endpoint a specified distance or direction? How does this affect the measure of the angle? Is this still considered a translation?



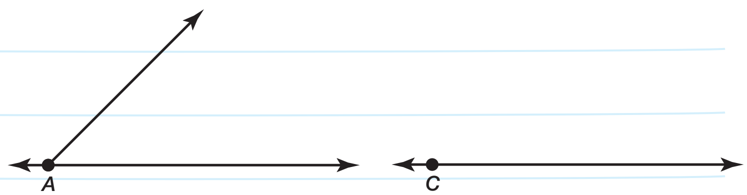
## PROBLEM 2 Constructing an Angle

Previously, you translated an angle on the coordinate plane using line segments that were associated with units of measure. You can also translate an angle not associated with units of measure.

This basic geometric construction to translate an angle not associated with units of measure is called **copying an angle** or **duplicating an angle**. The construction is performed using a compass and a straightedge.

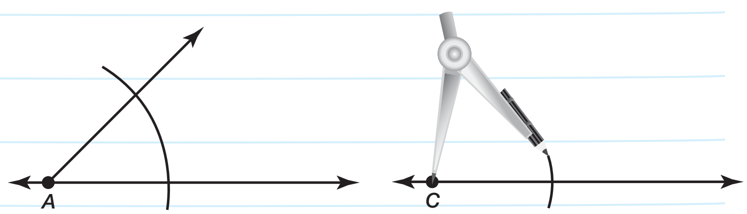
**Construct a Starter Line**

Use a straightedge to construct a starter line. Label point *C* on the new segment.



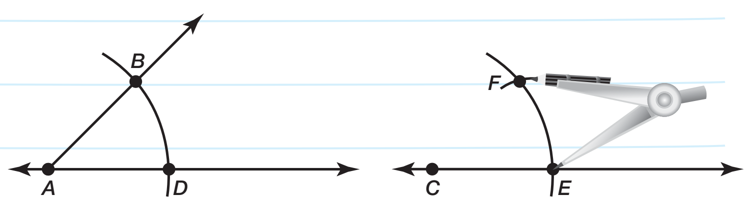

**Construct an Arc**

Construct an arc with center *A*. Using the same radius, construct an arc with center *C*.



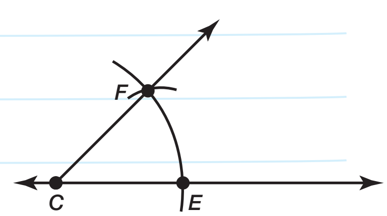

**Construct Another Arc**

Label points *B*, *D*, and *E*. Construct an arc with radius *BD* and center *E*. Label the intersection *F*.




**Construct a Ray**

Construct ray *CF*.
$\angle BAD \cong \angle FCE$.



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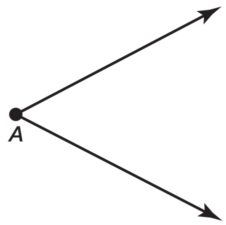
1. Construct an angle that is twice the measure of $\angle A$. Then explain how you performed the construction.



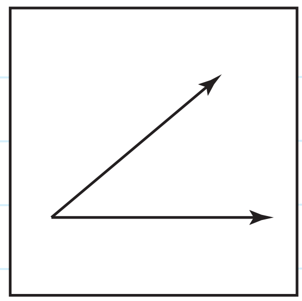

2. How is duplicating an angle similar to duplicating a line segment? How is it different?

## PROBLEM 3 Bisecting an Angle

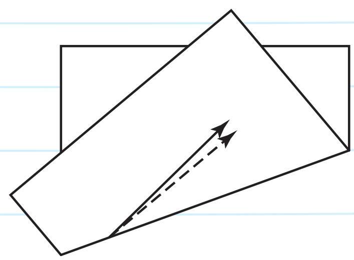
Just as line segments can be bisected, angles can be bisected too. If a ray is drawn through the vertex of an angle and divides the angle into two angles of equal measure, or two congruent angles, this ray is called an **angle bisector**. The construction used to create an angle bisector is called **bisecting an angle**.

One way to bisect an angle is using tracing paper.

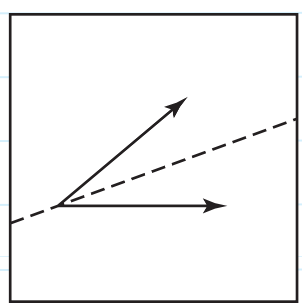
Draw an angle on the paper.

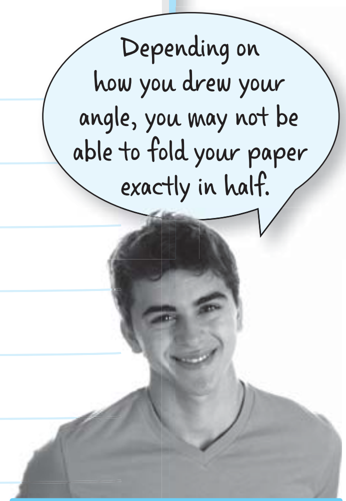


Fold the paper so that the rays lie on top of each other. The fold should pass through the vertex of the angle.



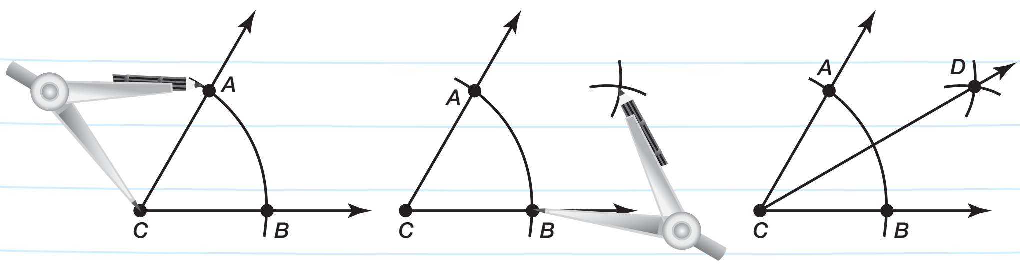
Open the paper. The crease represents the angle bisector.






1. Angela states that as long as the crease goes through the vertex, it is an angle bisector. Is she correct? Why or why not?

You can also bisect an angle using a compass and a straightedge.




**Construct an Arc**

Place the compass at *C*. Construct an arc that intersects both sides of the angle. Label the intersections *A* and *B*.

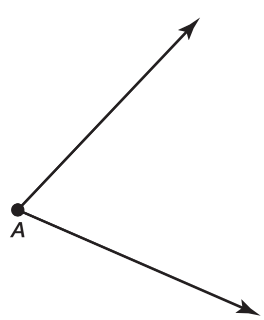
**Construct Another Arc**

Place the compass at *A*. Construct an arc. Then, place the compass point at *B*. Using the same radius, construct another arc.

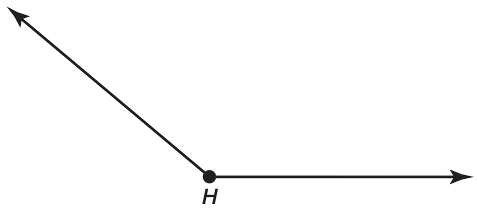
**Construct a Ray**

Label the intersection of the two arcs *D*. Use a straightedge to construct a ray through *C* and *D*. Ray *CD* bisects $\angle C$.

2. Construct the bisector of $\angle A$.

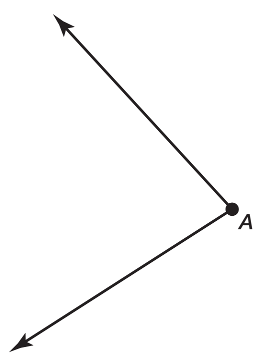


3. Construct an angle that is one-fourth the measure of ∠*H*. Explain how you performed the construction.




4. Describe how to construct an angle that is one-eighth the measure of ∠*H* from Question 3.

5. Use a compass and straightedge to show that the two angles formed by the angle bisector of angle *A* are congruent. Explain how you performed the construction.



## Talk the Talk

Translating an angle on the coordinate plane using coordinates and translating an angle by construction using construction tools both preserve the measure of the angle.

1. How are the two methods of translation similar?

2. How are the two methods of translation different?

3. Does either a vertical or a horizontal translation of an angle alter the measure of the angle? Explain why or why not.

Be prepared to share your solutions and methods.

# Did You Find a Parking Space?

1.5

## Parallel and Perpendicular Lines on the Coordinate Plane

### LEARNING GOALS

In this lesson, you will:

- Determine whether lines are parallel.
- Identify and write the equations of lines parallel to given lines.
- Determine whether lines are perpendicular.
- Identify and write the equations of lines perpendicular to given lines.
- Identify and write the equations of horizontal and vertical lines.
- Calculate the distance between a line and a point not on the line.

### KEY TERM

- point-slope form

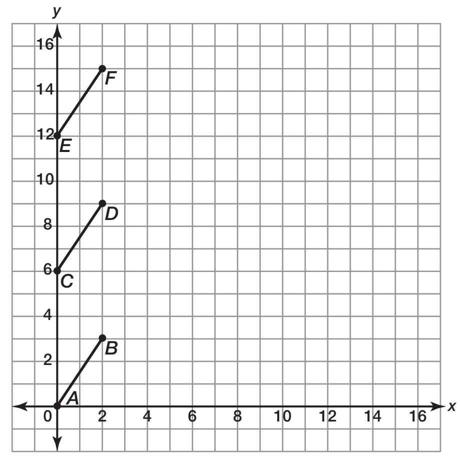
They seem simple enough, but parking lots require a great deal of planning. Transportation engineers use technology and science to plan, design, operate, and manage parking lots for many modes of transportation. During the planning stage of a parking lot, these engineers must keep in mind the needs of the facility that will use the parking lot as well as the needs of the drivers. Engineers must think about the entrances and exits as well as the surrounding streets and their traffic flow. Even the weather must be taken into account if the lot is being built somewhere with heavy rain or snow!

Only thinking about the cars and their drivers, what needs might affect an engineer's plans? What would make a parking lot "good" or "bad"? Can you think of anything else that might affect the planning of a parking lot other than the factors already mentioned?

## PROBLEM 1 Parking Spaces

Large parking lots have line segments painted to mark the locations where vehicles are supposed to park. The layout of these line segments must be considered carefully so that there is enough room for the vehicles to move and park in the lot without other vehicles being damaged.

The line segments shown model parking spaces in a parking lot. One grid square represents one square meter.




1. What do you notice about the line segments that form the parking spaces?

2. What is the vertical distance between $\overline{AB}$ and $\overline{CD}$ and between $\overline{CD}$ and $\overline{EF}$?

3. Carefully extend $\overline{AB}$ to create line $p$, extend $\overline{CD}$ to create line $q$, and extend $\overline{EF}$ to create line $r$.

4. Calculate the slope of each line. What do you notice?

The **point-slope form** of the equation of a line that passes through $(x_1, y_1)$ and has slope $m$ is $y - y_1 = m(x - x_1)$.

5. Use the point-slope form to write the equations of lines $p$, $q$, and $r$. Then, write the equations in slope-intercept form.

6. What do the $y$-intercepts tell you about the relationship between these lines in the problem situation?

7. If you were to draw $\overline{GH}$ above $\overline{EF}$ to form another parking space, predict what the slope and equation of the line will be without graphing the new line. How did you come to this conclusion?

8. Shawna and Lexi made the following statements about parallel lines.

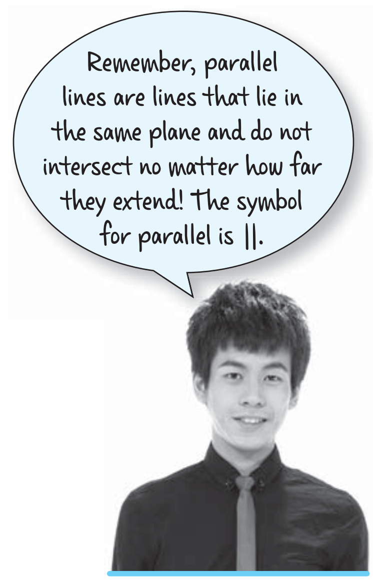
**Shawna**

*When you have parallel lines, all of their slopes are going to be equal!*

**Lexi**

*The y-intercepts of parallel lines are always a multiple of the same number!*

a. Explain why Shawna is correct.



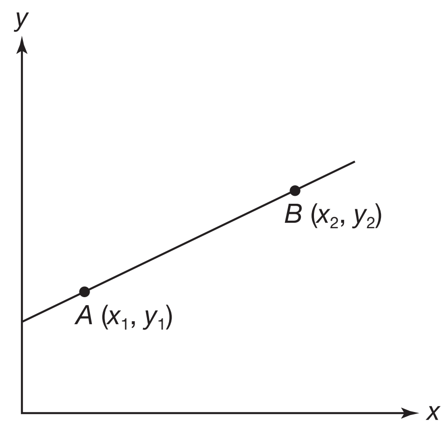

b. Provide a counter-example to show that Lexi is incorrect.

9. Write equations for three lines that are parallel to the line $y = -2x + 4$. Explain how you determined your equations.

10. Write an equation for the line that is parallel to the line $y = 5x + 3$ and passes through the point (4, 0). Explain how you determined your equation.

11. Without graphing the equations, predict whether the lines given by $y - 2x = 5$ and $2x - y = 4$ are parallel.

12. Consider the graph shown.




a. Use the graph to translate line segment *AB* up *a* units.

b. Identify the *x*- and *y*-coordinates of each corresponding point on the image.

c. Use the slope formula to calculate the slope of the pre-image.

d. Use the slope formula to calculate the slope of the image.

e. How does the slope of the image compare to the slope of the pre-image?

f. How would you describe the relationship between the graph of the image and the graph of the pre-image?

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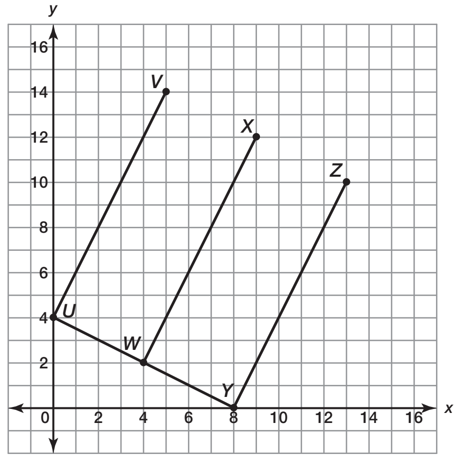
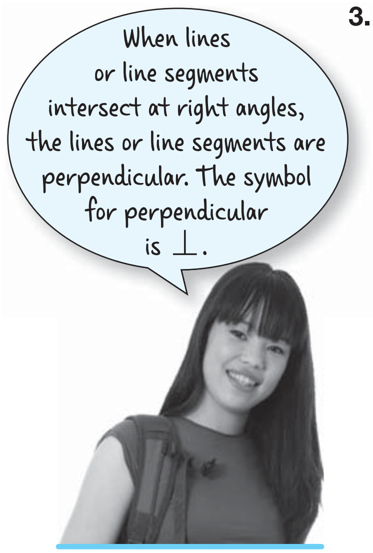
## PROBLEM 2 More Parking Spaces

The line segments shown represent parking spaces in a truck stop parking lot. One grid square represents one square meter.




1. Use a protractor to determine the measures of $\angle VUW$, $\angle XWY$, and $\angle ZYW$. What similarity do you notice about the angles?


2. Carefully extend $\overline{UY}$ to create line $p$, extend $\overline{UV}$ to create line $q$, extend $\overline{WX}$ to create line $r$, and extend $\overline{YZ}$ to create line $s$ on the coordinate plane.




3. Determine whether each set of lines are perpendicular or parallel. Then predict how the slopes of the lines will compare. Do not actually calculate the slopes of the lines when you make your prediction.

   a. *q*, *r*, *s*

   b. *p* and *q*

   c. *p* and *r*

   d. *p* and *s*

4. Calculate the slopes of lines *p*, *q*, *r*, and *s*.

5. Determine the product of the slopes of two perpendicular lines. Use lines *p*, *q*, *r*, and *s* to provide an example.

6. Describe the difference between the slopes of two parallel lines and the slopes of two perpendicular lines.

1

When the product of two numbers is 1, the numbers are reciprocals of one another. When the product of two numbers is $-1$, the numbers are negative reciprocals of one another. So the slopes of perpendicular lines are negative reciprocals of each other.

7. Do you think that the $y$-intercepts of perpendicular lines tell you anything about the relationship between the perpendicular lines? Explain your reasoning.

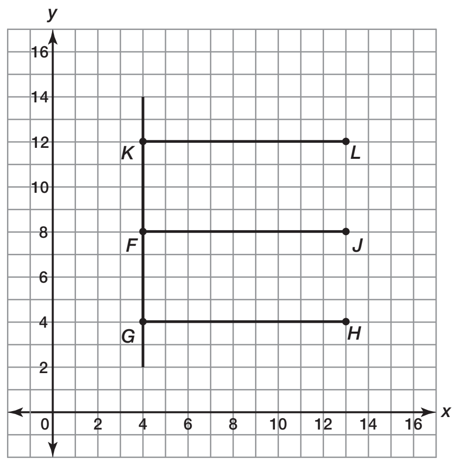
8. Write equations for three lines that are perpendicular to the line $y = -2x + 4$. Explain how you determined your equations.

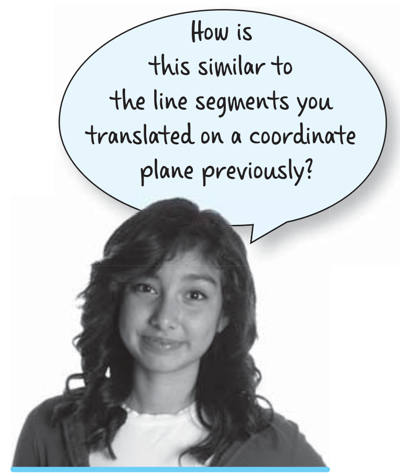
9. Write an equation for the line that is perpendicular to the line $y = 5x + 3$ and passes through the point (4, 0). Show all your work and explain how you determined your equation.

10. Without graphing the equations, determine whether the lines $y + 2x = 5$ and $2x - y = 4$ are perpendicular. Explain how you determined your answer.

## PROBLEM 3 Horizontal and Vertical

Consider the graph shown.




1. Carefully extend $\overline{GK}$ to create line $p$, extend $\overline{GH}$ to create line $q$, extend $\overline{FJ}$ to create line $r$, and extend $\overline{KL}$ to create line $s$.

2. Consider the three horizontal lines you drew for Question 1. For any horizontal line, if $x$ increases by one unit, by how many units does $y$ change?

3. What is the slope of any horizontal line? Does this make sense? Why or why not?

4. Consider the vertical line you drew for Question 1. Suppose that $y$ increases by one unit. By how many units does $x$ change?

5. What is the slope of any vertical line? Does this make sense? Explain why or why not.

6. Determine whether each statement is always, sometimes, or never true. Explain your reasoning.

   a. Vertical lines are ___________________ parallel.

   b. Horizontal lines are ___________________ parallel.

7. Describe the relationship between any vertical line and any horizontal line. Explain your reasoning.

8. Write an equation for a horizontal line and an equation for a vertical line that pass through the point $(2, -1)$.

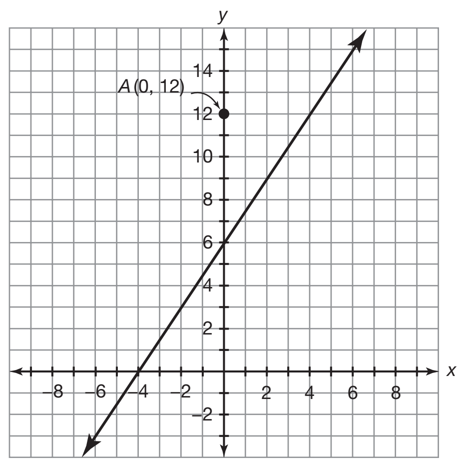
9. Write an equation for a line that is perpendicular to the line $x = 5$ and passes through the point $(1, 0)$.

10. Write an equation for a line that is perpendicular to the line $y = -2$ and passes through the point $(5, 6)$.

## PROBLEM 4 Distance Between Lines and Points

1. Describe the shortest distance between a point and a line.

2. The equation of the line shown on the coordinate plane is $f(x) = \frac{3}{2}x + 6$. Draw the shortest segment between the line and the point $A$ (0, 12). Label the point where the segment intersects $f(x)$ as point $B$.




3. What information do you need in order to calculate the length of $\overline{AB}$ using the Distance Formula?

4. How can you calculate the intersection point of $\overline{AB}$ and the line $f(x) = \frac{3}{2}x + 6$ algebraically?

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5. Calculate the distance from point $A$ to the line $f(x) = \frac{3}{2}x + 6$.

a. Write an equation for $\overline{AB}$.

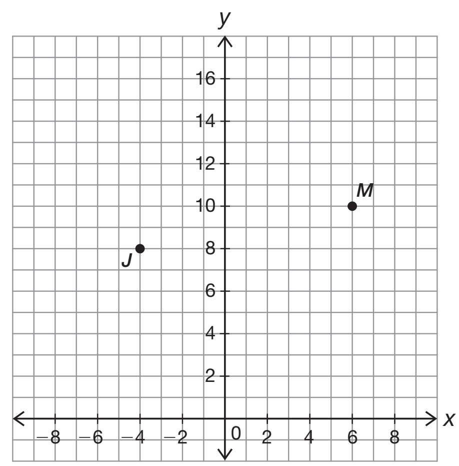
b. Calculate the point of intersection of $\overline{AB}$ and the line $f(x) = \frac{3}{2}x + 6$.

c. Calculate the length of $\overline{AB}$.

d. What is the distance from point $A$ to the line $f(x) = \frac{3}{2}x + 6$?

## PROBLEM 5 Where's the School?

Molly's house is located at point *M* (6, 10). Jessica's house is located 2 blocks south and 10 blocks west of Molly's house at point *J*. Create a graph showing where Molly and Jessica live. Molly and Jessica live the same distance from their school. Use algebra to describe all possible locations of the school.



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## Talk the Talk

1. Consider the linear equation $6x - 2y - 5 = 0$.

   Decide which of the following lines are parallel, perpendicular, or neither to the given line. Explain your reasoning.

   **a.** $y + 3x = \frac{5}{2}$

   **b.** $12x - 4y = 8$

   **c.** $3y = -x + 5$

   **d.** $y = \frac{1}{3}x - 5$

   **e.** $12x - 10 = 4y$

   **f.** $\frac{2}{3}x + 2y = 8$

2. What do you notice about the slopes of the lines perpendicular to the given line?

3. How would you best describe the relationship of all lines perpendicular to the given line?

Be prepared to share your solutions and methods.

# Making Copies—Just as Perfect as the Original!

1.6

## Constructing Perpendicular Lines, Parallel Lines, and Polygons

### LEARNING GOALS

In this lesson, you will:

- Construct a perpendicular line to a given line.
- Construct a parallel line to a given line through a point not on the line.
- Construct an equilateral triangle given the length of one side of the triangle.
- Construct an isosceles triangle given the length of one side of the triangle.
- Construct a square given the perimeter (as the length of a given line segment).
- Construct a rectangle that is not a square given the perimeter (as the length of a given line segment).

### KEY TERM

- perpendicular bisector

### CONSTRUCTIONS

- a perpendicular line to a given line through a point on the line
- a perpendicular line to a given line through a point not on the line

There's an old saying that you might have heard before: "They broke the mold when they made me!" A person says this to imply that they are unique. Of course, humans do not come from molds, but there are plenty of things that do.

For example, take a look at a dime if you have one handy. Besides some tarnish on the coin and the year the coin was produced, it is identical to just about every other dime out there. Creating and duplicating a coin a few billion times is quite a process involving designing the coin, creating multiple molds (and negatives of the molds), cutting the design onto metal, and on and on.

Can you think of any times when the "original" might be more important than a duplicate? Can you think of any examples where the "original" product might be more expensive than a generic brand of the same product?

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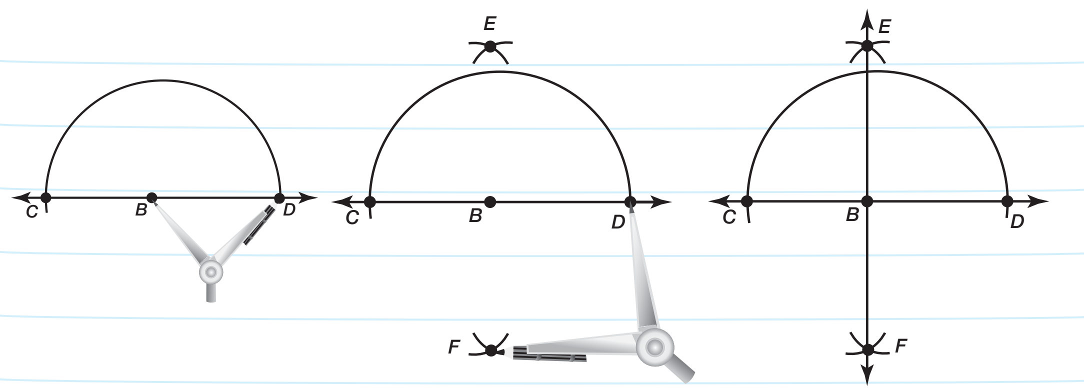
## PROBLEM 1 Constructing Perpendicular Lines

Previously, you practiced bisecting a line segment and locating the midpoint of a line segment by construction. In fact, you were also constructing a line segment perpendicular to the original line segment.

A **perpendicular bisector** is a line, line segment, or ray that bisects a line segment and is also perpendicular to the line segment.

Follow the steps to construct a perpendicular line through a point on the line.




**Construct an Arc**

Use *B* as the center and construct an arc. Label the intersections points *C* and *D*.

**Construct Other Arcs**

Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

**Construct a Line**

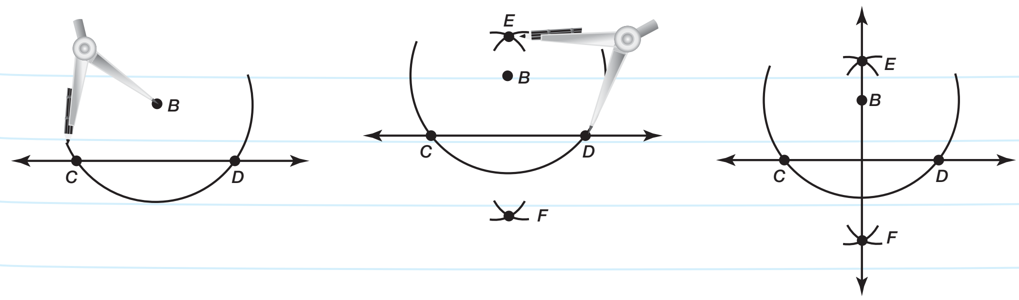
Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to line *CD*.

1. Construct a line perpendicular to the given line through point *P*.


2. How is constructing a segment bisector and constructing a perpendicular line through a point on a line different?

3. Do you think that you can only construct a perpendicular line through a point that is on a line? Why or why not?

Follow these steps to construct a perpendicular line through a point not on a line.




**Construct an Arc**

Use *B* as the center and construct an arc. Label the intersection points *C* and *D*.

**Construct Other Arcs**

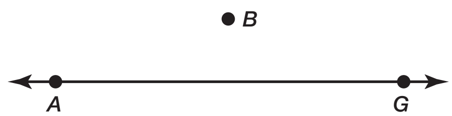
Open the compass larger than the radius. Use *C* and *D* as centers and construct arcs above and below the line. Label the intersection points *E* and *F*.

**Construct a Line**

Use a straightedge to connect points *E* and *F*. Line *EF* is perpendicular to line *CD*.

**4.** Amos claims that it is only possible to construct a perpendicular line through horizontal and vertical lines because the intersection of the points must be right angles. Loren claims that a perpendicular line can be constructed through any line and any point on or not on the line. Who is correct? Correct the rationale of the student who is *not* correct.

**5.** Construct a line perpendicular to $\overleftrightarrow{AG}$ through point *B*.




**6.** How is the construction of a perpendicular line through a point on a line different from the construction of a perpendicular line through a point not on the line?

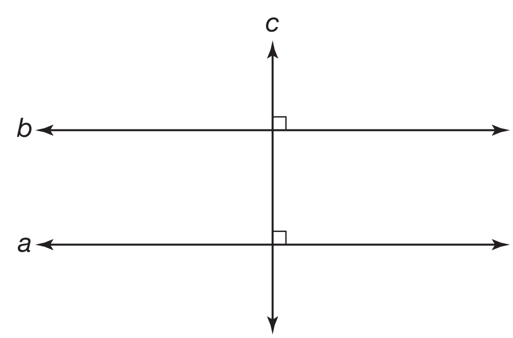
**7.** Choose a point on the perpendicular bisector of $\overline{AG}$ and measure the distance from your point to point *A* and point *G*. Choose another point on the perpendicular bisector and measure the distance from this point to point *A* and point *G*. What do you notice.

**8.** Make a conjecture about the distance from any point on a perpendicular bisector to the endpoints of the original segment.

## PROBLEM 2 Constructing Parallel Lines

To construct a line parallel to a given line, you must use a perpendicular line.

1. Analyze the figure shown.




Describe the relationship between the lines given.

a. *a* and *c*

b. *b* and *c*

c. *a* and *b*

**2.** Construct line *e* parallel to line *d*. Then, describe the steps you performed for the construction.

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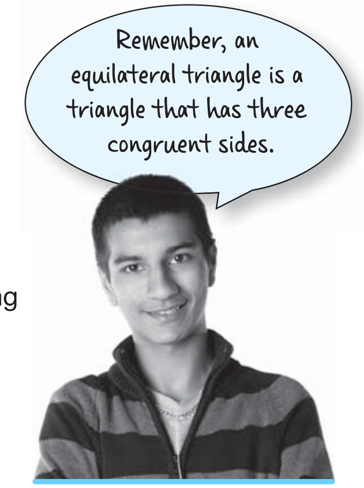
## PROBLEM 3 Constructing an Equilateral Triangle




In the rest of this lesson, you will construct an equilateral triangle, an isosceles triangle, a square, and a rectangle that is not a square. To perform the constructions, use only a compass and straightedge and rely on the basic geometric constructions you have learned such as duplicating a line segment, duplicating an angle, bisecting a line segment, bisecting an angle, constructing perpendicular lines, and constructing parallel lines.

**1.** The length of one side of an equilateral triangle is shown.



**a.** What do you know about the other two sides of the equilateral triangle you will construct given the line segment shown?

**b.** Construct an equilateral triangle using the given side length. Then, describe the steps you performed for the construction.

**2.** Sophie claims that she can construct an equilateral triangle by duplicating the line segment three times and having the endpoints of all three line segments intersect. Roberto thinks that Sophie's method will not result in an equilateral triangle. Who is correct? Explain why the incorrect student's rationale is not correct.

## PROBLEM 4 Constructing an Isosceles Triangle


1. The length of one side of an isosceles triangle that is not an equilateral triangle is shown.



   a. Construct an isosceles triangle that is *not* an equilateral triangle using the given side length. Then, describe the steps you performed for the construction.

   b. Explain how you know your construction resulted in an isosceles triangle that is not an equilateral triangle.

   c. How does your construction compare to your classmates' constructions?

## PROBLEM 5 Constructing a Square Given the Perimeter

Now you will construct a square using a given perimeter.

1. The perimeter of a square is shown by $\overline{AB}$.


   a. Construct the square. Then, describe the steps that you performed for the construction.

   b. How does your construction compare to your classmates' constructions?

## PROBLEM 6 Constructing a Rectangle Given the Perimeter

**1.** The perimeter of a rectangle is shown by $\overline{AB}$.


**a.** Construct the rectangle that is not a square. Then, describe the steps you performed for the construction.

**b.** How does this construction compare to your classmates' constructions?

Be prepared to share your solutions and methods.

# What's the Point?

## Points of Concurrency

1.7

**LEARNING GOALS**

In this lesson, you will:

- Construct the incenter, circumcenter, centroid, and orthocenter.
- Locate points of concurrency using algebra.

**KEY TERMS**

- concurrent
- point of concurrency
- circumcenter
- incenter
- median
- centroid
- altitude
- orthocenter

Imagine playing a game of darts with a lot of people. You're all aiming for the bullseye, but of course it is the most difficult spot to hit on the dartboard. Therefore, it is probably the least hit spot on the board. Maybe you play a few hundred rounds, with each person having three throws on each turn, and each hit—no matter where it is—is recorded.

Now imagine that the dartboard is taken away, but all of the hits are shown as little dots on an empty wall. Could you determine the bullseye location using just the locations of those dots? How might you do it?

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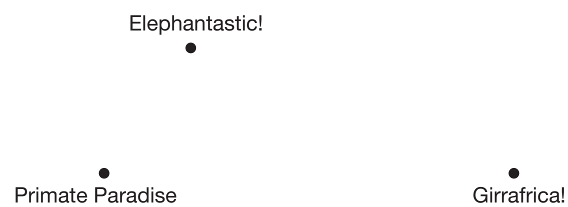
## PROBLEM 1 The New Zoo Review

Josh and Lezlee have been given the task of building an information kiosk for the zoo. They want the kiosk to be easily accessible from each of the various exhibits. Three of the exhibits at the zoo are shown.




1. Josh suggests building the kiosk so that it is equal distance from Giraffrica! and Primate Paradise.
   a. Lezlee replies that the only location of the kiosk can be determined by determining the midpoint of the line segment between the two exhibits.

      Is Lezlee correct? Explain your reasoning.

   b. Use a construction to show all possible locations of the kiosk so that it is equal distance from Giraffrica! and Primate Paradise. Explain your reasoning.

2. Lezlee suggests building the kiosk so that it is equal distance from Elephantastic! and Primate Paradise. Use a construction to show all possible locations of the kiosk so that it is equal distance from Elephantastic! and Primate Paradise. Explain your reasoning.

3. Josh then wonders where the kiosk could be built so that it is equal distance from Elephantastic! and Giraffrica!. Use a construction to show all possible locations of the kiosk so that it is equal distance from Elephantastic! and Giraffrica!. Explain your reasoning.

4. Describe how to determine a location that is the same distance from all three exhibits. Is there more than one possible location that is equidistant from all three exhibits? Explain your reasoning.

5. Verify that the location you described in Question 4 is equidistant from each exhibit.

## PROBLEM 2 Concurrence

**Concurrent** lines, rays, or line segments are three or more lines, rays, or line segments intersecting at a single point. The **point of concurrency** is the point at which concurrent lines, rays, or segments intersect.

1. Draw three concurrent lines and label *C* as the point of concurrency.

2. Draw three concurrent rays and label *C* as the point of concurrency.

3. Draw three concurrent line segments and label *C* as the point of concurrency.

## PROBLEM 3 Investigating the Circumcenter

1. Make a conjecture about the perpendicular bisectors of an acute triangle by performing the following steps.

   a. Draw an acute triangle that is not an equilateral triangle.

   b. Construct the three perpendicular bisectors of your acute triangle.

   c. Make a conjecture about the intersection of the three perpendicular bisectors of an acute triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

1

2. Make a conjecture about the perpendicular bisectors of an obtuse triangle by performing the following steps.

   a. Draw an obtuse triangle.

   b. Construct the three perpendicular bisectors of your obtuse triangle.

   c. Make a conjecture about the intersection of the perpendicular angle bisectors of an obtuse triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

3. Make a conjecture about the perpendicular bisectors of a right triangle by performing the following steps.

   a. Draw a right triangle.

   b. Construct the three perpendicular bisectors of your right triangle.

   c. Make a conjecture about the intersection of the three perpendicular bisectors of a right triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

4. Make a conjecture about the perpendicular bisectors of an equilateral triangle by performing the following steps.

   a. Construct an equilateral triangle.

   b. Construct the three perpendicular bisectors of your equilateral triangle.

   c. Make a conjecture about the intersection of the three perpendicular bisectors of an equilateral triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

5. Make a conjecture about the intersection of the three perpendicular bisectors of any triangle. Is the intersection on the interior, exterior, or on the triangle?

The **circumcenter** is the point of concurrency of the three perpendicular bisectors of a triangle.

**6.** Consider the four triangles that you drew in Questions 1 through 4.

**a.** Measure the distance from the circumcenter to each vertex of the triangle.

**b.** Is the circumcenter always, sometimes, or never equidistant from each vertex of the triangle? Explain your reasoning.

**c.** Measure the distance from the circumcenter to each side of the triangle.

**d.** Is the circumcenter always, sometimes, or never equidistant from each side of the triangle? Explain your reasoning.

## PROBLEM 4 Investigating the Incenter

1. Make a conjecture about the angle bisectors of an acute triangle by performing the following steps.

   a. Draw an acute triangle that is not an equilateral triangle.

   b. Construct the three angle bisectors of your acute triangle.

   c. Make a conjecture about the intersection of the three angle bisectors of an acute triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

2. Make a conjecture about the angle bisectors of an obtuse triangle by performing the following steps.

   a. Draw an obtuse triangle.

   b. Construct the three angle bisectors of your obtuse triangle.

   c. Make a conjecture about the intersection of the three angle bisectors of an obtuse triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

3. Make a conjecture about the angle bisectors of a right triangle by performing the following steps.

   a. Draw a right triangle.

   b. Construct the three angle bisectors of your right triangle.

   c. Make a conjecture about the intersection of the three angle bisectors of a right triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

4. Make a conjecture about the angle bisectors of an equilateral triangle by performing the following steps.

   a. Construct an equilateral triangle.

   b. Construct the three angle bisectors of your equilateral triangle.

   c. Make a conjecture about the intersection of the three angle bisectors of an equilateral triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

5. Make a conjecture about the intersection of the three angle bisectors of any triangle. Is the intersection on the interior, exterior, or on the triangle?

1

The **incenter** is the point of concurrency of the three angle bisectors of a triangle.

6. Consider the four triangles that you drew in Questions 1 through 4.

   a. Measure the distance from the incenter to each vertex of the triangle.

   b. Is the incenter always, sometimes, or never equidistant from each vertex of the triangle? Explain your reasoning.

   c. Measure the distance from the incenter to each side of the triangle.

   d. Is the incenter always, sometimes, or never equidistant from each side of the triangle? Explain your reasoning.

## PROBLEM 5 Investigating the Centroid

A **median** of a triangle is a line segment that connects a vertex to the midpoint of the opposite side.

1. Make a conjecture about the medians of an acute triangle by performing the following steps.

   **a.** Draw an acute triangle that is not an equilateral triangle.

   **b.** Construct the three medians of your acute triangle.

   **c.** Make a conjecture about the intersection of the three medians of an acute triangle.

   **d.** Compare your conjecture to the conjectures of your classmates. What do you notice?

2. Make a conjecture about the medians of an obtuse triangle by performing the following steps.

   a. Draw an obtuse triangle.

   b. Construct the three medians of your obtuse triangle.

   c. Make a conjecture about the intersection of the three medians of an obtuse triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

3. Make a conjecture about the medians of a right triangle by performing the following steps.

   a. Draw a right triangle.

   b. Construct the three medians of your right triangle.

   c. Make a conjecture about the intersection of the three medians of a right triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

1

4. Make a conjecture about the medians of an equilateral triangle by performing the following steps.

   a. Construct an equilateral triangle.

   b. Construct the three medians of your equilateral triangle.

   c. Make a conjecture about the intersection of the three medians of an equilateral triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

5. Make a conjecture about the intersection of the three medians of any triangle. Is the intersection on the interior, exterior, or on the triangle?

The **centroid** is the point of concurrency of the three medians of a triangle.



**6.** Consider the four triangles that you drew in Questions 1 through 4.

**a.** Measure the distance from the centroid to each vertex of the triangle.

**b.** Is the centroid always, sometimes, or never equidistant from each vertex of the triangle? Explain your reasoning.

**c.** Measure the distance from the centroid to each side of the triangle.

**d.** Is the centroid always, sometimes, or never equidistant from each side of the triangle? Explain your reasoning.

**7.** The centroid divides each median into two segments. Compare the distance from the centroid to the vertex and the distance from the centroid to the midpoint of the opposite side in each of the triangles you created. What is the ratio?

## PROBLEM 6 Investigating the Orthocenter

An **altitude** of a triangle is a line segment that is perpendicular to a side of the triangle and has one endpoint at the opposite vertex.

1. Make a conjecture about the altitudes of an acute triangle by performing the following steps.

   **a.** Draw an acute triangle that is not an equilateral triangle.

   **b.** Construct the three altitudes of your acute triangle.

   **c.** Make a conjecture about the intersection of the three altitudes of an acute triangle.

   **d.** Compare your conjecture to the conjectures of your classmates. What do you notice?

2. Make a conjecture about the altitudes of an obtuse triangle by performing the following steps.

   a. Draw an obtuse triangle.

   b. Construct the three altitudes of your obtuse triangle.

   c. Make a conjecture about the intersection of the three altitudes of an obtuse triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

3. Make a conjecture about the altitudes of a right triangle by performing the following steps.

   a. Draw a right triangle.

   b. Construct the three altitudes of your right triangle.

   c. Make a conjecture about the intersection of the three altitudes of a right triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

4. Make a conjecture about the altitudes of an equilateral triangle by performing the following steps.

   a. Construct an equilateral triangle.

   b. Construct the three altitudes of your equilateral triangle.

   c. Make a conjecture about the intersection of the three altitudes of an equilateral triangle.

   d. Compare your conjecture to the conjectures of your classmates. What do you notice?

5. Make a conjecture about the intersection of the three altitudes of any triangle. Is the intersection on the interior, exterior, or on the triangle?

The **orthocenter** is the point of concurrency of the three altitudes of a triangle.

**6.** Consider the four triangles that you drew in Questions 1 through 4.

**a.** Measure the distance from the orthocenter to each vertex of the triangle.

**b.** Is the orthocenter always, sometimes, or never equidistant from each vertex of the triangle? Explain your reasoning.

**c.** Measure the distance from the orthocenter to each side of the triangle.

**d.** Is the orthocenter always, sometimes, or never equidistant from each side of the triangle? Explain your reasoning.

## PROBLEM 7 Points of Concurrency

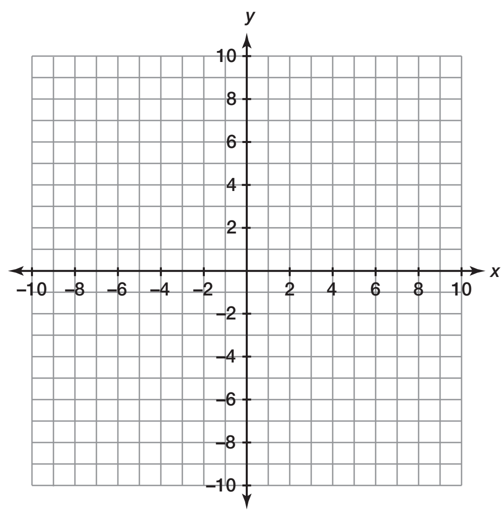
1. Examine the incenter, circumcenter, centroid, and orthocenter for the equilateral triangles you created. What do you notice?

2. Complete the table to describe the location of each point of currency for acute, obtuse, and right triangles.

| | Acute Triangle | Obtuse Triangle | Right Triangle |
|---|---|---|---|
| **Incenter** | | | |
| **Circumcenter** | | | |
| **Centroid** | | | |
| **Orthocenter** | | | |

## PROBLEM 8 Using Algebra with Points of Concurrency

1. Form a triangle on the grid by connecting the points $A(-6, -8)$, $B(6, -8)$, and $C(0, 10)$.




2. Classify triangle $ABC$ by performing the following steps.

   **a.** Calculate the length of each side of the triangle using the Distance Formula.

   **b.** Use the side lengths to classify the triangle. Explain your reasoning.

3. Describe how you could use algebra to calculate the coordinates of each point of concurrency.

   a. centroid

   b. circumcenter

   c. orthocenter


   d. incenter

1

4. Calculate the centroid of triangle *ABC*.

a. Calculate the midpoint of each side of the triangle.

b. Write an equation for each median.

c. Calculate the coordinates of the centroid.

d. Plot the centroid on the grid in Question 1.

1

5. Calculate the circumcenter of triangle *ABC*.

   a. Calculate the midpoint of each side of the triangle.

   b. Calculate the slope of each side of the triangle.

   c. Write an equation for each perpendicular bisector.

**d.** Calculate the coordinates of the circumcenter.

**e.** Plot the circumcenter on the grid in Question 1.

**6.** Calculate the orthocenter of triangle *ABC*.

**a.** Calculate the slope of each side of the triangle.

**b.** Write an equation for each altitude.

**c.** Calculate the coordinates of the orthocenter.

**d.** Plot the orthocenter on the grid in Question 1.

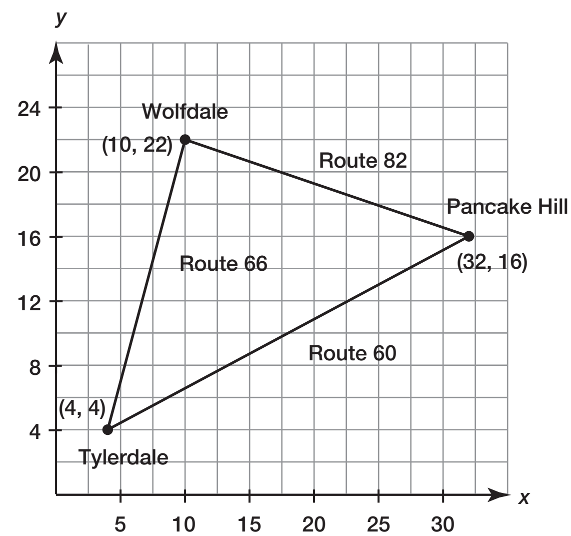
**7.** What do you notice about the location of the centroid, circumcenter, and orthocenter for isosceles triangle *ABC*?

**8.** Do you think the behavior that you described in Question 8 will also be true for the incenter? Explain your reasoning.

**9.** Do you think the behavior that you described in Questions 7 through 8 will be true for a triangle that is not isosceles? Explain how you could test your theory.

## PROBLEM 9 The Mall

A construction company plans to build a large mall to serve three small towns as shown on the map below. They are considering a location that is equidistant from each of the three towns and agree to pay for building new roads connecting the mall to the three towns. Cost for building new roads is $150,000 per mile. Each unit on the graph represents one mile.




1. What point of concurrency is most useful in this situation? Explain your reasoning.

2. Determine the approximate coordinates of the location of the mall.

3. Determine the approximate distance from the mall location to each town. Round to the nearest mile.

4. How much money should be budgeted for road construction in this project?

## Talk the Talk

1. Determine which point of concurrency would be most helpful in each situation.

   a. A flea market is situated on a triangular piece of land. Each entrance is located at one of the three vertices of triangle. Joanie wants to set up her merchandise at a location that is equidistant from all three entrances.

   b. An artist is building a mobile with several metal triangles of various sizes. The triangles are connected to each other using steel rods and the rods are welded onto each triangle at a point which would allow the triangle to balance horizontally.

   c. Jim's backyard is a triangular plot of land. He is using fencing to build a circular dog pen. He wants the dog pen to be as large as possible and needs to determine the location of the center of the circular dog pen.

2. The intersection of the three altitudes of any triangle is best described by this point.

3. The point of concurrency located two-thirds the way from the vertex to the midpoint of the opposite side and otherwise known as the center of gravity.



Be prepared to share your solutions and methods.

# Chapter 1 Summary

## KEY TERMS

- point (1.1)
- line (1.1)
- collinear points (1.1)
- plane (1.1)
- compass (1.1)
- straightedge (1.1)
- sketch (1.1)
- draw (1.1)
- construct (1.1)
- coplanar lines (1.1)
- skew lines (1.1)
- ray (1.1)
- endpoint of a ray (1.1)
- line segment (1.1)
- endpoints of a line segment (1.1)
- congruent line segments (1.1)
- Distance Formula (1.2)
- transformation (1.2)
- rigid motion (1.2)
- translation (1.2)
- pre-image (1.2)
- image (1.2)
- arc (1.2)
- midpoint (1.3)
- Midpoint Formula (1.3)
- segment bisector (1.3)
- angle (1.4)
- angle bisector (1.4)
- point-slope form (1.5)
- perpendicular bisector (1.6)
- concurrent (1.7)
- point of concurrency (1.7)
- circumcenter (1.7)
- incenter (1.7)
- median (1.7)
- centroid (1.7)
- altitude (1.7)
- orthocenter (1.7)

## CONSTRUCTIONS

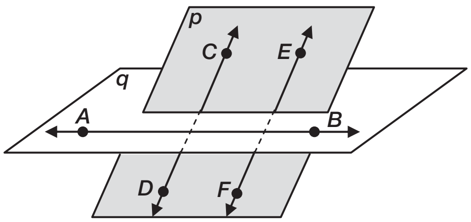
- copying a line segment (1.2)
- duplicating a line segment (1.2)
- bisecting a line segment (1.3)
- copying an angle (1.4)
- duplicating an angle (1.4)
- bisecting an angle (1.4)
- perpendicular line to a given line through a point on the line (1.6)
- perpendicular line to a given line through a point not on the line (1.6)

1

## 1.1 Identifying Points, Lines, and Planes

A point is a location in space that has no size or shape. A line is a straight continuous arrangement of an infinite number of points. A plane is a flat surface that has an infinite length and width, but no depth. Collinear points are points that are located on the same line. Coplanar lines are two or more lines that are located in the same plane. Skew lines are two or more lines that are not in the same plane.

**Example**




Points *A* and *B* lie on $\overline{AB}$, points *C* and *D* lie on $\overline{CD}$, and points *E* and *F* lie on $\overline{EF}$.

Line *AB* lies in plane *q*. Lines *CD* and *EF* lie in plane *p*.

Points *A* and *B* are collinear. Points *C* and *D* are collinear. Points *E* and *F* are collinear.

Lines *CD* and *EF* are coplanar.

Lines *AB* and *CD* are skew. Lines *AB* and *EF* are skew.

Planes *p* and *q* intersect.

## 1.2 Applying the Distance Formula

The Distance Formula can be used to calculate the distance between two points on the coordinate plane. The Distance Formula states that if $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the coordinate plane, then the distance $d$ between $(x_1, y_1)$ and $(x_2, y_2)$ is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

**Example**

Calculate the distance between the points (3, −2) and (−5, 1).

$x_1 = 3, y_1 = -2, x_2 = -5, y_2 = 1$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-5 - 3)^2 + [1 - (-2)]^2}$$
$$= \sqrt{(-8)^2 + (3)^2}$$
$$= \sqrt{64 + 9}$$
$$= \sqrt{73}$$
$$\approx 8.5$$

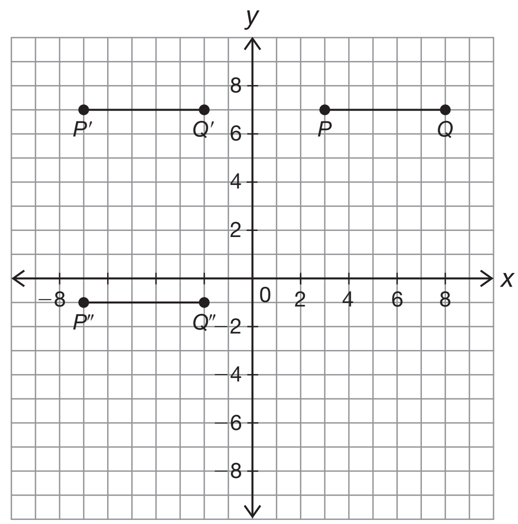
The distance between the points (3, −2) and (−5, 1) is $\sqrt{73}$ units, or approximately 8.5 units.

## 1.2 Translating Line Segments on the Coordinate Plane

A translation is a rigid motion that slides each point of a figure the same distance and direction. A horizontal translation of a line segment on the coordinate plane changes the *x*-coordinates of both endpoints while leaving the *y*-coordinates the same. A vertical translation changes the *y*-coordinates of both endpoints while leaving the *x*-coordinates the same.

### Example

Line segment *PQ* is translated horizontally 10 units to the left to create $\overline{P'Q'}$. Line segment *P′Q′* is translated vertically 8 units down to create line segment $\overline{P''Q''}$.



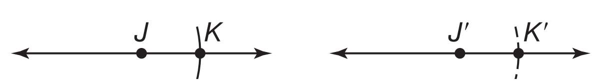

| Line Segment | $\overline{PQ}$ | $\overline{P'Q'}$ | $\overline{P''Q''}$ |
|---|---|---|---|
| Coordinates of Endpoints | (3, 7)<br>(8, 7) | (−7, 7)<br>(−2, 7) | (−7, −1)<br>(−2, −1) |

The lengths of the images and the pre-images remain the same after each translation.

## 1.2 Duplicating a Line Using Construction Tools

A straightedge and compass can be used to duplicate a line.

### Example




Line segment *JK* can be duplicated using a straightedge and compass by drawing a starter line and then duplicating a line segment that is the same length as $\overline{JK}$.

1

## 1.3 Applying the Midpoint Formula

A midpoint is a point that is exactly halfway between two given points. The Midpoint Formula can be used to calculate the coordinates of a midpoint. The Midpoint Formula states that if $(x_1, y_1)$ and $(x_2, y_2)$ are two points on the coordinate plane, then the midpoint of the line segment that joins these two points is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

**Example**

Calculate the midpoint of a line segment with the endpoints (−8, −3) and (4, 6).

$x_1 = -8, y_1 = -3, x_2 = 4, y_2 = 6$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) = \left(\frac{-8 + 4}{2}, \frac{-3 + 6}{2}\right)$$

$$= \left(\frac{-4}{2}, \frac{3}{2}\right)$$

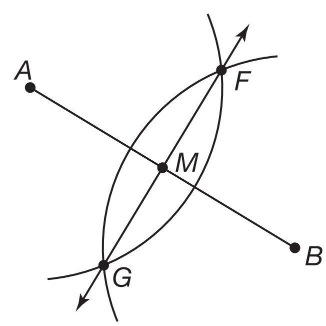
$$= \left(-2, \frac{3}{2}\right)$$

The midpoint of the line segment is $\left(-2, \frac{3}{2}\right)$.

## 1.3 Bisecting a Line Segment Using Construction Tools

Construction tools can be used to bisect a line segment.

**Example**



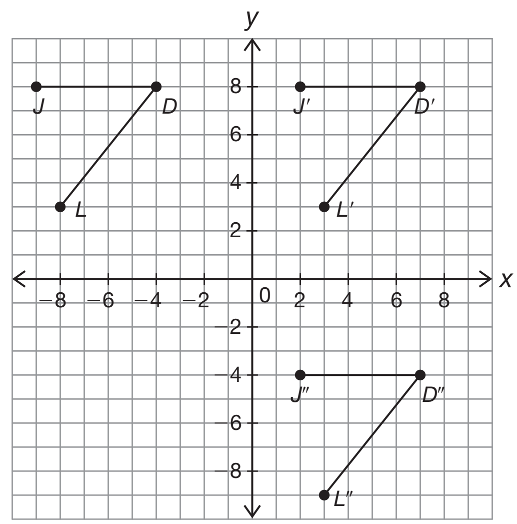

Open the radius of the compass to more than half the length of the original line segment. Construct an arc using one endpoint as the center. Keeping the compass at the same radius, construct an arc using the other endpoint as center. Label and connect the points created by the intersection of the arcs. Line segment *FG* bisects $\overline{AB}$.

## 1.4 Translating an Angle on the Coordinate Plane

Translating an angle on the coordinate plane is a rigid motion that slides the angle, either horizontally or vertically, on the coordinate plane. Because it is a rigid motion, the angle measures of the image and the pre-image are the same. Horizontal translations only impact the *x*-coordinates of the endpoints; vertical translations only impact the *y*-coordinates of the endpoints.

### Example

Angle *JDL* is translated horizontally 11 units right to form $\angle J'D'L'$. Angle $J'D'L'$ is translated vertically 12 units down to create $\angle J''D''L''$.




| **Line Segment** | $\overline{JD}$ | $\overline{J'D'}$ | $\overline{J''D''}$ |
|---|---|---|---|
| **Coordinates of Endpoints** | (−9, 8)<br>(−4, 8) | (2, 8)<br>(7, 8) | (2, −4)<br>(7, −4) |

| **Line Segment** | $\overline{DL}$ | $\overline{D'L'}$ | $\overline{D''L''}$ |
|---|---|---|---|
| **Coordinates of Endpoints** | (−4, 8)<br>(−8, 3) | (7, 8)<br>(3, 3) | (7, −4)<br>(3, −9) |

The measure of the angle images and pre-images remain the same after each translation.

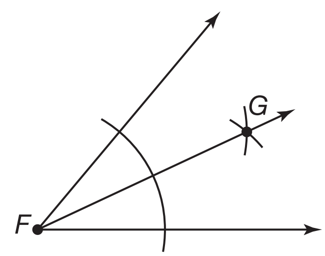
1

## 1.4 Bisecting an Angle Using Construction Tools

An angle bisector is a ray drawn through the vertex of an angle that divides the angle into two angles of equal measure.

### Example

Angle $F$ can be bisected using construction tools.




Place the compass on the vertex of the angle. Construct an arc that intersects both sides of the angle. Place the compass at one of the intersection points and construct an arc, then using the same radius of the compass construct an arc using the other intersection point. Construct a ray connecting the vertex to the intersection of the arcs. Ray $FG$ bisects $\angle F$.

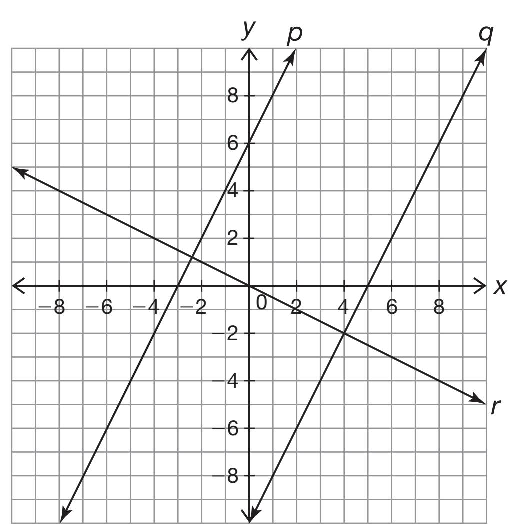
## 1.5 Determining Whether Lines Are Parallel or Perpendicular

The slopes of parallel lines are equal. The slopes of perpendicular lines are negative reciprocals of each other. When the product of two numbers is $-1$, the numbers are negative reciprocals of each other.

### Example

The equation of line $p$ is $y = 2x + 6$, the equation of line $q$ is $y = 2x - 10$, and the equation of line $r$ is $y = -\frac{1}{2}x$. The slopes of lines $p$ and $q$ are equal, so lines $p$ and $q$ are parallel.

The slopes of lines $p$ and $r$ are negative reciprocals of each other, so lines $p$ and $r$ are perpendicular. Also, the slopes of lines $q$ and $r$ are negative reciprocals of each other, so lines $q$ and $r$ are also perpendicular.



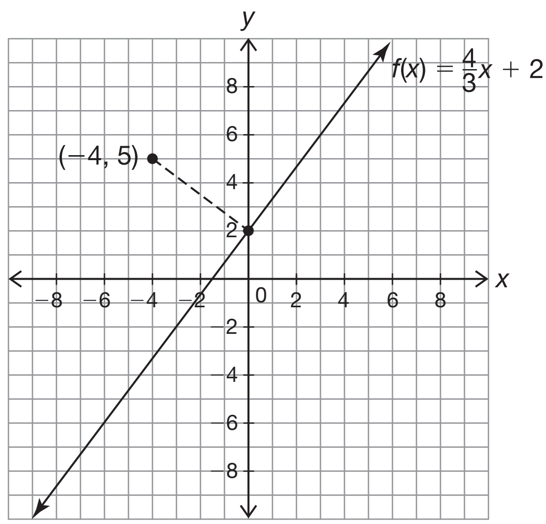
## Determining the Distance Between Lines and Points



The shortest distance between any line and a point not on that line is the length of the perpendicular segment drawn from the point to the line. The shortest distance between a line and a point not on the line can be determined using the equation of the perpendicular segment drawn from the point to the line, the equation of the original line, and the Distance Formula.

### Example

Calculate the distance between the line given by the equation $f(x) = \frac{4}{3}x + 2$ and the point (−4, 5).




Equation of perpendicular segment:

$y = mx + b$

$5 = -\frac{3}{4}(-4) + b$

$5 = 3 + b$

$2 = b$

$y = -\frac{3}{4}x + 2$

Point of intersection:

$\frac{4}{3}x + 2 = -\frac{3}{4}x + 2$

$16x = -9x$

$25x = 0$

$x = 0$

$y = \frac{4}{3}(0) + 2$

$y = 2$

(0, 2)

Distance between point of intersection and given point:

$x_1 = 0, y_1 = 2, x_2 = -4, y_2 = 5$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-4 - 0)^2 + (5 - 2)^2}$

$= \sqrt{(-4)^2 + (3)^2}$

$= \sqrt{16 + 9}$

$= \sqrt{25}$

$= 5$

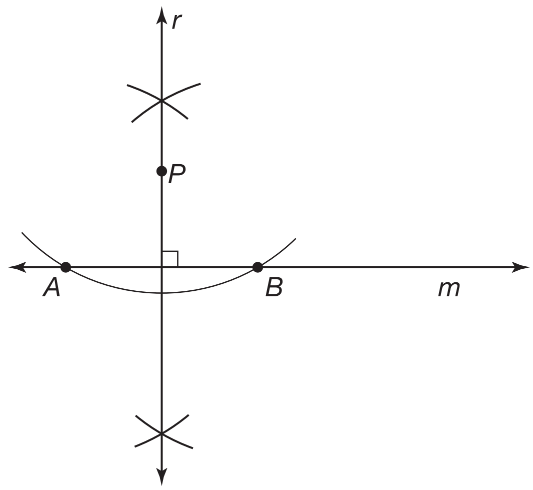
The distance between the line given by the equation $f(x) = \frac{4}{3}x + 2$ and the point (−4, 5) is 5 units.

1

## 1.6 Constructing Perpendicular Lines

Perpendicular lines can be constructed through a given point using construction tools.

**Example**



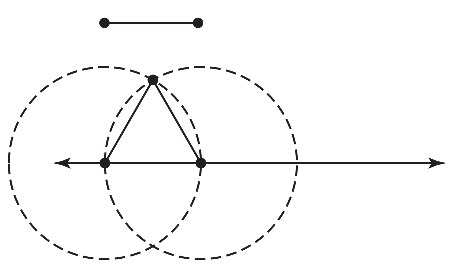

Use the given point *P* as the center and construct an arc that passes through the given line. Open the compass radius. Construct an arc above and below the given line using one of the intersection points just created. Keeping the radius the same, construct an arc above and below the given line using the other intersection point. Connect the intersection points of the arcs which should also pass through the given point. Line *r* is perpendicular to line *m*.

## 1.6 Constructing Equilateral Triangles

Equilateral triangles have 3 congruent sides. Construction tools can be used to construct an equilateral triangle given the length of one side.

**Example**

Construct an equilateral triangle with the side length shown.



Construct a starter line and duplicate the given segment onto the starter line. Construct a circle using an endpoint of the line segment as the center. Then construct another circle using the other endpoint as the center. Connect the point of intersection of the circles to each endpoint using line segments.

1.6

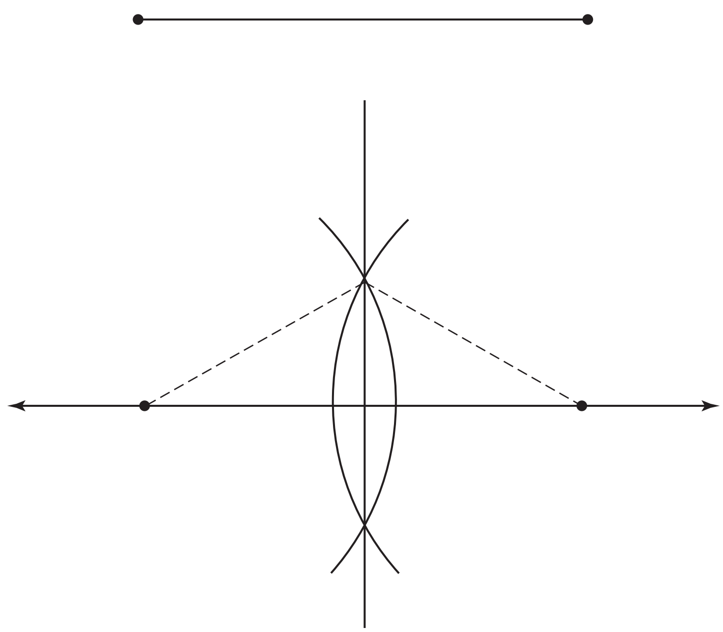
## Constructing Isosceles Triangles

1

An isosceles triangle is a triangle that has at least two sides of equal length.

**Example**

Construct an isosceles triangle with the side length shown.



Construct a starter line and duplicate the given line segment. Then construct a perpendicular bisector through the line segment. Connect the endpoints of each line segment to a point on the bisector.

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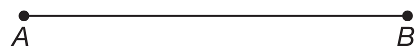
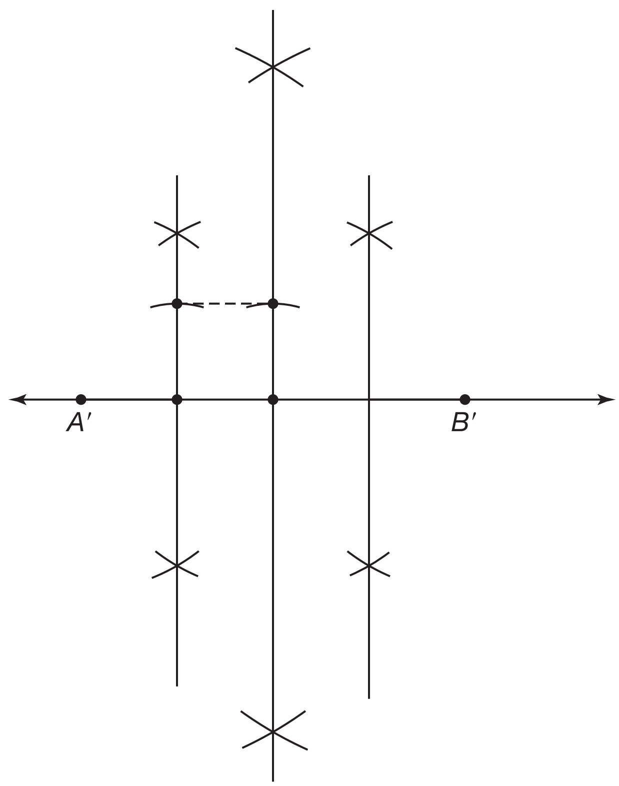
## 1.6 Constructing Squares

A square can be constructed using construction tools.

**Example**

Construct a square using the perimeter given.



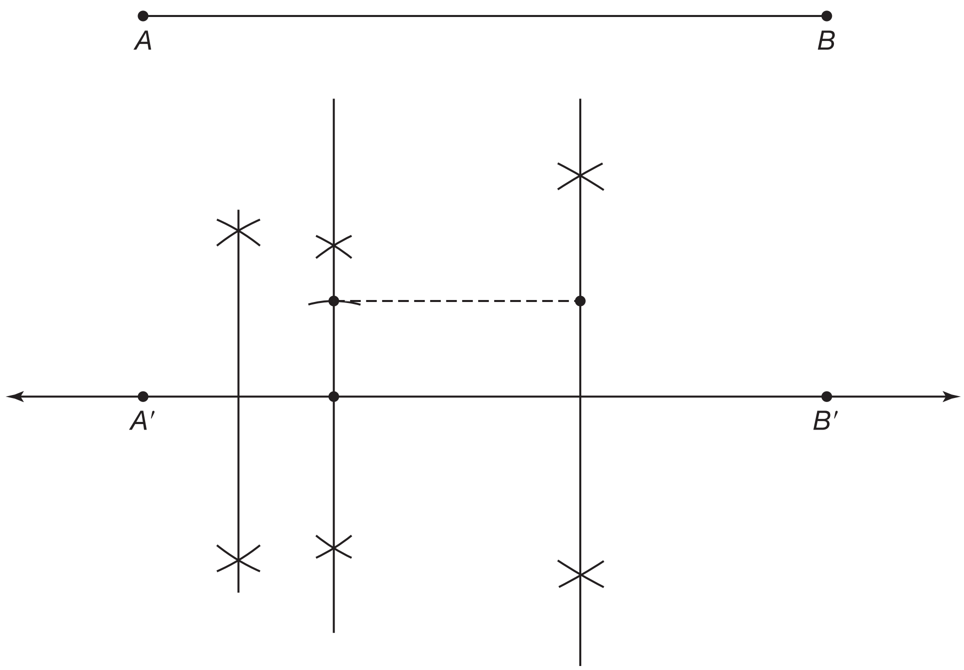

Construct a starter line and duplicate the given perimeter. Bisect the line segment using a perpendicular bisector. Then, bisect each of the created line segments to create 4 line segments of equal length. Duplicate one of the line segments along two perpendicular bisectors to create the height of the square. Connect the two endpoints of the line segments representing the height to complete the square.

## 1.6 Constructing Rectangles That Are Not Squares

A rectangle can be constructed in a similar method to constructing a square using a given perimeter of the rectangle.

### Example

Construct a rectangle using the perimeter given.




Construct a starter line and duplicate the given perimeter. Place a point anywhere on the line segment except in the middle dividing the line segment into two unequal line segments. Then, draw perpendicular bisectors through each of the line segments to create four line segments. Choose one of the line segments to use as the base of the rectangle. Duplicate another line segment that is not the same size as the base on two of the perpendicular bisectors to use as the height of the rectangle. Finally, connect the endpoints of the line segments representing the height to create a rectangle.

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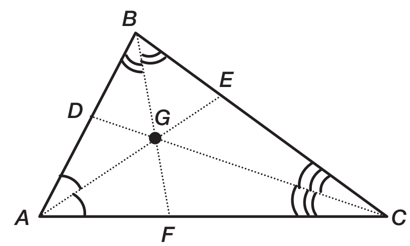
# 1.7 Identifying Points of Concurrency

When three or more lines intersect at the same point, the lines are called concurrent lines. The point at which the concurrent lines intersect is called the point of concurrency.

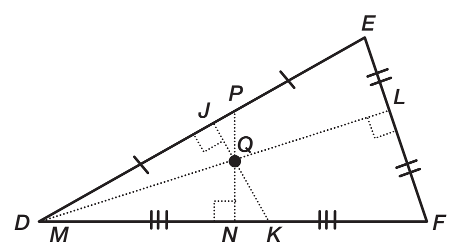
There are special types of points of concurrency in triangles. The incenter of a triangle is the point at which the three angle bisectors of a triangle are concurrent. The circumcenter of a triangle is the point at which the three perpendicular bisectors of a triangle are concurrent. The centroid is the point at which the three medians of a triangle are concurrent. The orthocenter is the point at which the three altitudes of a triangle are concurrent.

### Examples

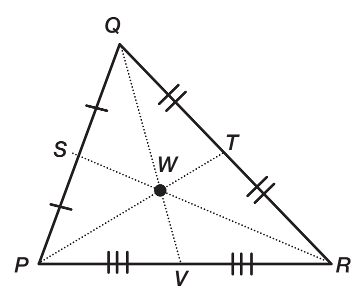
In $\triangle ABC$, $\overline{AE}$, $\overline{BF}$, and $\overline{CD}$ are angle bisectors. So, point $G$ is the incenter, and $DG = EG = FG$.




In $\triangle DEF$, $\overline{JK}$, $\overline{LM}$, and $\overline{NP}$ are perpendicular bisectors. So, point $Q$ is the circumcenter, and the distances from the circumcenter to each vertex are the same.




In $\triangle PQR$, $\overline{PT}$, $\overline{QV}$, and $\overline{RS}$ are medians. So, point $W$ is the centroid, and $PW = 2TW$, $QW = 2VW$, and $RW = 2SW$.




In $\triangle XYZ$, $\overline{XB}$, $\overline{YC}$, and $\overline{ZA}$ are altitudes. So, point $D$ is the orthocenter.