Introduction to Proof

2

A Little Dash of Logic

2.1

Foundations for Proof

LEARNING GOALS

In this lesson, you will:

- Define inductive and deductive reasoning.
- Identify methods of reasoning.
- Compare and contrast methods of reasoning.
- Create examples using inductive and deductive reasoning.
- Identify the hypothesis and conclusion of a conditional statement.
- Explore the truth values of conditional statements.
- Use a truth table.

KEY TERMS

- induction
- deduction
- counterexample
- conditional statement
- propositional form
- propositional variables
- hypothesis
- conclusion
- truth value
- truth table

One of the most famous literary detectives is Sherlock Holmes. Created by author Sir Arthur Conan Doyle, Sherlock Holmes first appeared in print in 1887 in the novel *A Study in Scarlet*. The character has gone on to appear in four novels, 56 short stories, and over 200 films. The Guinness Book of World Records lists Holmes as the most portrayed movie character, with more than 70 different actors playing the part.

Holmes is most famous for his keen powers of observation and logical reasoning, which always helped him solve the case. In many literary and film adaptations, Holmes is known to remark, "Elementary, my dear Watson," after explaining to his assistant how he solved the mystery. However, this well-known phrase doesn't actually appear in any of the stories written by Doyle. It first appeared in the 1915 novel *Psmith, Journalist* by P.G. Wodehouse and also appeared at the end of the 1929 film *The Return of Sherlock Holmes*. Regardless, this phrase will probably always be associated with the famous detective.

PROBLEM 1 How Do You Figure?

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1. Emma considered the following statements.

   - $4^2 = 4 \times 4$
   - Nine cubed is equal to nine times nine times nine.
   - 10 to the fourth power is equal to four factors of 10 multiplied together.

   Emma concluded that raising a number to a power is the same as multiplying the number as many times as indicated by the exponent. How did Emma reach this conclusion?

2. Ricky read that raising a number to a power is the same as multiplying that number as many times as indicated by the exponent. He had to determine seven to the fourth power using a calculator. So, he entered $7 \times 7 \times 7 \times 7$. How did Ricky reach this conclusion?

3. Compare Emma's reasoning to Ricky's reasoning.

4. Jennifer is a writing consultant. She is paid $900 for a ten-hour job and $1980 for a twenty-two-hour job.

   a. How much does Jennifer charge per hour?

   b. To answer Question 4, part (a), did you start with a general rule and make a conclusion, or did you start with specific information and create a general rule?

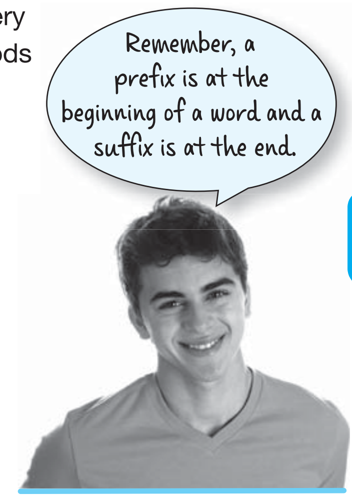
5. Your friend Aaron tutors elementary school students. He tells you that the job pays $8.25 per hour.

   a. How much does Aaron earn from working 4 hours?

   b. To answer Question 5, part (a), did you start with a general rule and make a conclusion, or did you start with specific information and create a general rule?

PROBLEM 2 Is This English Class or Algebra?

The ability to use information to reason and make conclusions is very important in life and in mathematics. There are two common methods of reasoning. You can construct the name for each method of reasoning using your knowledge of prefixes, root words, and suffixes.



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Word Fragment	Prefix, Root Word, or Suffix	Meaning
in-	Prefix	*toward* or *up to*
de-	Prefix	*down from*
-duc-	Root Word	*to lead* and often *to think*, from the Latin word *duco*
-tion	Suffix	*the act of*

1. Form a word that means "the act of thinking down from."

2. Form a word that means "the act of thinking toward or up to."

Induction is reasoning that uses specific examples to make a conclusion. Sometimes you will make generalizations about observations or patterns and apply these generalizations to new or unfamiliar situations. For example, you may notice that when you don't study for a test, your grade is lower than when you do study for a test. You apply what you learned from these observations to the next test you take.

Deduction is reasoning that uses a general rule to make a conclusion. For example, you may learn the rule for which direction to turn a screwdriver: "righty tighty, lefty loosey." If you want to remove a screw, you apply the rule and turn the screwdriver counterclockwise.

3. Consider the reasoning used by Emma, Ricky, Jennifer, and Aaron in Problem 1.

   a. Who used inductive reasoning?

   b. Who used deductive reasoning?

PROBLEM 3 Coming to Conclusions

A problem situation can provide you with a great deal of information that you can use to make conclusions. It is important to identify specific and general information in a problem situation to reach appropriate conclusions. Some information may be irrelevant to reach the appropriate conclusion.

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Ms. Ross teaches an Economics class every day from 1:00 PM to 2:15 PM. Students' final grade is determined by class participation, homework, quizzes, and tests. She noticed that Andrew has not turned in his homework 3 days this week. She is concerned that Andrew's grade will fall if he does not turn in his homework.

Irrelevant Information:
Ms. Ross teaches an Economics class every day from 1:00 PM to 2:15 PM.

General information:
Students' final grade is determined by class participation, homework, quizzes, and tests.

Specific information:
Andrew has not turned in his homework 3 days this week.

Conclusion:
Andrew's grade will fall if he does not turn in his homework.

1. Did Ms. Ross use induction or deduction to make this conclusion? Explain your answer.

2. Conner read an article that claimed that tobacco use greatly increases the risk of getting cancer. He then noticed that his neighbor Matilda smokes. Conner is concerned that Matilda has a high risk of getting cancer.

   a. Which information is specific and which information is general in this problem situation?

   b. What is the conclusion in this problem?

   c. Did Conner use inductive or deductive reasoning to make the conclusion? Explain your reasoning.

   d. Is Conner's conclusion correct? Explain your reasoning.

3. Molly returned from a trip to England and tells you, “It rains every day in England!” She explains that it rained each of the five days she was there.

a. Which information is specific and which information is general in this problem situation?

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b. What is the conclusion in this problem?

c. Did Molly use inductive or deductive reasoning to make the conclusion? Explain your answer.

d. Is Molly’s conclusion correct? Explain your reasoning.

4. Dontrell takes detailed notes in history class and math class. His classmate Trang will miss biology class tomorrow to attend a field trip. Trang's biology teacher asks him if he knows someone who always takes detailed notes. Trang tells his biology teacher that Dontrell takes detailed notes. Trang's biology teacher suggests that Trang should borrow Dontrell's notes because he concludes that Dontrell's notes will be detailed.

   a. What conclusion did Trang make? What information supports this conclusion?

   b. What type of reasoning did Trang use? Explain your reasoning.

   c. What conclusion did the biology teacher make? What information supports this conclusion?

   d. What type of reasoning did the biology teacher use? Explain your reasoning.

   e. Will Trang's conclusion always be true? Will the biology teacher's conclusion always be true? Explain your reasoning.

5. The first four numbers in a sequence are 4, 15, 26, and 37.

   a. What is the next number in the sequence? How did you calculate the next number?

   b. Describe how you used both induction and deduction, and what order you used these reasonings to make your conclusion.

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6. The first three numbers in a sequence are 1, 4, 9 . . . Marie and Jose both determined that the fourth number in the sequence is 16. Marie's rule involved multiplication whereas Jose's rule involved addition.

   a. What types of reasoning did Marie and Jose use to determine the fourth number in the sequence?

   b. What rule did Marie use to determine the fourth number in the sequence?

   c. What rule did Jose use to determine the fourth number in the sequence?

   d. Who used the correct rule? Explain your reasoning.

PROBLEM 4 Why Is This False?

There are two reasons why a conclusion may be false. Either the assumed information is false, or the argument is not valid.

1. Derek tells his little brother that it will not rain for the next 30 days because he "knows everything." Why is this conclusion false?

2. Two lines are not parallel, so the lines must intersect. Why is this conclusion false?

3. Write an example of a conclusion that is false because the assumed information is false.

4. Write an example of a conclusion that is false because the argument is not valid.

To show that a statement is false, you can provide a *counterexample*. A **counterexample** is a specific example that shows that a general statement is not true.

5. Provide a counterexample for each of these statements to demonstrate that they are not true.

   a. All prime numbers are odd.

   b. The sum of the measures of two acute angles is always greater than 90°.

PROBLEM 5 You Can't Handle the Truth Value

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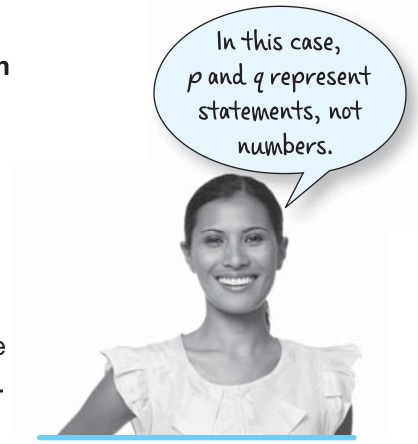
A **conditional statement** is a statement that can be written in the form "If *p*, then *q*." This form is the **propositional form** of a conditional statement. It can also be written using symbols as $p \rightarrow q$, which is read as "*p* implies *q*." The variables *p* and *q* are **propositional variables**. The **hypothesis** of a conditional statement is the variable *p*. The **conclusion** of a conditional statement is the variable *q*.

The **truth value** of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then the truth value of the statement is considered true. The truth value of a conditional statement is either true or false, but not both.



You can identify the hypothesis and conclusion from a conditional statement.

Conditional Statement

If $x^2 = 36$, then $x = 6$ or $x = -6$.

Hypothesis of the Conditional Statement

$x^2 = 36$

Conclusion of the Conditional Statement

$x = 6$ or $x = -6$.

Consider the conditional statement: If the measure of an angle is 32°, then the angle is acute.

1. What is the hypothesis *p*?

2. What is the conclusion *q*?

3. If *p* is true and *q* is true, then the truth value of a conditional statement is true.

a. What does the phrase "If *p* is true" mean in terms of the conditional statement?

b. What does the phrase "If *q* is true" mean in terms of the conditional statement?

c. Explain why the truth value of the conditional statement is true if both *p* and *q* are true.

4. If p is true and q is false, then the truth value of a conditional statement is false.

   a. What does the phrase "If p is true" mean in terms of the conditional statement?

   b. What does the phrase "If q is false" mean in terms of the conditional statement?

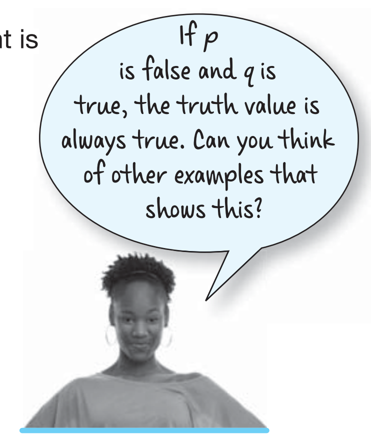
   c. Explain why the truth value of the conditional statement is false if p is true and q is false.

5. If p is false and q is true, then the truth value of a conditional statement is true.

   a. What does the phrase "If p is false" mean in terms of the conditional statement?

   b. What does the phrase "If q is true" mean in terms of the conditional statement?

   c. Explain why the truth value of the conditional statement is true if p is false and q is true.



6. If p is false and q is false, then the truth value of a conditional statement is true.

   a. What does the phrase "If p is false" mean in terms of the conditional statement?

   b. What does the phrase "If q is false" mean in terms of the conditional statement?

   c. Explain why the truth value of the conditional statement is true if both p and q are false.

A **truth table** is a table that summarizes all possible truth values for a conditional statement $p \rightarrow q$. The first two columns of a truth table represent all possible truth values for the propositional variables p and q. The last column represents the truth value of the conditional statement $p \rightarrow q$.

The truth values for the conditional statement "If the measure of an angle is 32°, then the angle is acute" is shown.

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The truth value of the conditional statement $p \rightarrow q$ is determined by the truth value of p and the truth value of q.

- If p is true and q is true, then $p \rightarrow q$ is true.
- If p is true and q is false, then $p \rightarrow q$ is false.
- If p is false and q is true, then $p \rightarrow q$ is true.
- If p is false and q is false, then $p \rightarrow q$ is true.

p	q	$p \rightarrow q$
the measure of an angle is 32°	**the angle is acute**	**If the measure of an angle is 32°, then the angle is acute.**
T	T	T
T	F	F
F	T	T
F	F	T

7. Consider the conditional statement: If $m\overline{AB} = 6$ inches and $m\overline{BC} = 6$ inches, then $\overline{AB} \cong \overline{BC}$.

   a. What is the hypothesis p?

   b. What is the conclusion q?

   c. If both p and q are true, what does that mean? What is the truth value of the conditional statement if both p and q are true?

d. If *p* is true and *q* is false, what does that mean? What is the truth value of the conditional statement if *p* is true and *q* is false?

e. If *p* is false and *q* is true, what does that mean? What is the truth value of the conditional statement if *p* is false and *q* is true?

f. If both *p* and *q* are false, what does that mean? What is the truth value of the conditional statement if both *p* and *q* are false?

g. Summarize your answers to parts (a) through (f) by completing a truth table for the conditional statement.

p	***q***	$p \rightarrow q$

PROBLEM 6 Rewriting Conditional Statements

For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

1. If $\overrightarrow{BD}$ bisects $\angle ABC$, then $\angle ABD \cong \angle CBD$.

   Given:

   Prove:

2. $\overline{AM} \cong \overline{MB}$, if M is the midpoint of $\overline{AB}$.

   Given:

   Prove:

3. If $\overleftrightarrow{AB} \perp \overrightarrow{CD}$ at point C, then $\angle ACD$ is a right angle and $\angle BCD$ is a right angle.

   Given:

   Prove:

4. $\overleftrightarrow{WX}$ is the perpendicular bisector of $\overline{PR}$, if $\overleftrightarrow{WX} \perp \overline{PR}$ and $\overleftrightarrow{WX}$ bisects $\overline{PR}$.

Given:

Prove:

5. Mr. David wrote the following information on the board.

If $\overline{AC} \cong \overline{BC}$, then C is the midpoint of $\overline{AB}$.

He asked his students to discuss the truth of this conditional statement.

Susan said she believed the statement to be true in all situations. Marcus disagreed with Susan and said that the statement was not true all of the time.

What is Marcus thinking and who is correct?

Talk the Talk

1. Write a short note to a friend explaining induction and deduction. Include definitions of both terms and examples that are very easy to understand.

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Be prepared to share your solutions and methods.

2.2

And Now From a New Angle

Special Angles and Postulates

LEARNING GOALS

In this lesson, you will:

- Calculate the complement and supplement of an angle.
- Classify adjacent angles, linear pairs, and vertical angles.
- Differentiate between postulates and theorems.
- Differentiate between Euclidean and non-Euclidean geometries.

KEY TERMS

- supplementary angles
- complementary angles
- adjacent angles
- linear pair
- vertical angles
- postulate
- theorem
- Euclidean geometry
- Linear Pair Postulate
- Segment Addition Postulate
- Angle Addition Postulate

A compliment is an expression of praise, admiration, or congratulations. Often when someone does something noteworthy, you may "pay them a compliment" to recognize the person's accomplishments.

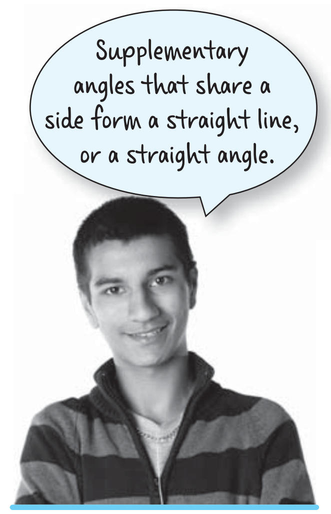
Even though they are spelled similarly, the word "complement" means something very different. To complement something means to complete or to make whole. This phrase is used in mathematics, linguistics, music, and art. For example, complementary angles have measures that sum to 180 degrees—making the straight angle "whole." In music, a complement is an interval that when added to another spans an octave—makes it "whole."

The film *Jerry McGuire* features the famous line "You complete me," meaning that the other person complements them or that together they form a whole. So, a complement can be quite a compliment indeed!

PROBLEM 1 Supplements and Complements

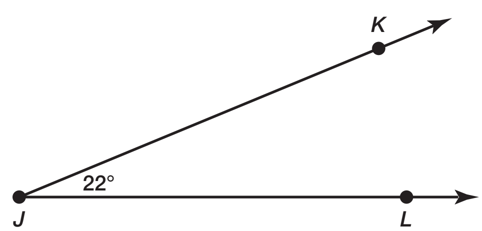
Two angles are **supplementary angles** if the sum of their angle measures is equal to 180°.

1. Use a protractor to draw a pair of supplementary angles that share a common side, and then measure each angle.



2. Use a protractor to draw a pair of supplementary angles that do not share a common side, and then measure each angle.

3. Calculate the measure of an angle that is supplementary to $\angle KJL$.

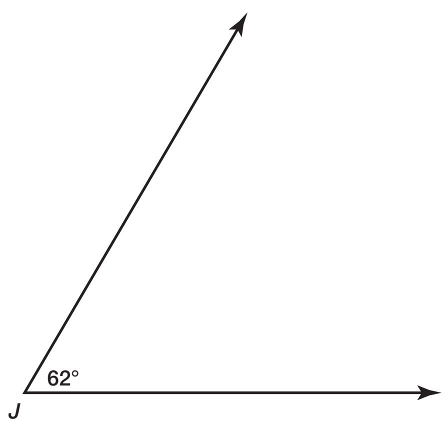


Two angles are **complementary angles** if the sum of their angle measures is equal to 90°.

4. Use a protractor to draw a pair of complementary angles that share a common side, and then measure each angle.

5. Use a protractor to draw a pair of complementary angles that do not share a common side, and then measure each angle.

6. Calculate the measure of an angle that is complementary to $\angle J$.



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7. Determine the measure of each angle. Show your work and explain your reasoning.

   a. Two angles are congruent and supplementary.

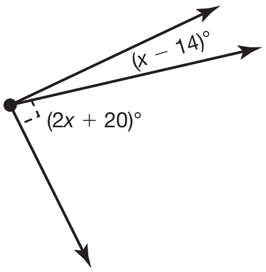
   b. Two angles are congruent and complementary.

   c. The complement of an angle is twice the measure of the angle.

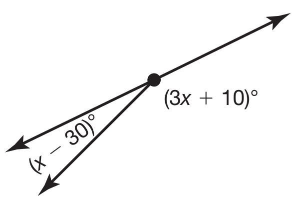
   d. The supplement of an angle is half the measure of the angle.

8. Determine the angle measures in each diagram.

a.



b.



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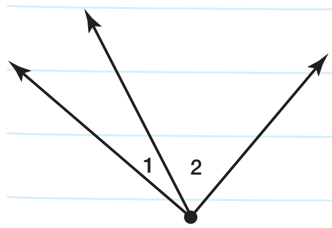
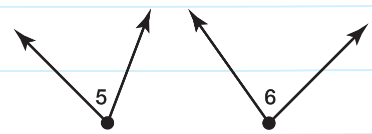
PROBLEM 2 Angle Relationships

You have learned that angles can be supplementary or complementary. Let's explore other angle relationships.

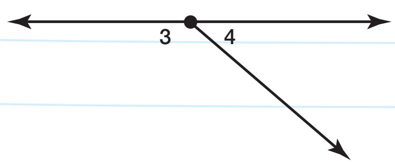
$\angle 1$ and $\angle 2$ are adjacent angles.



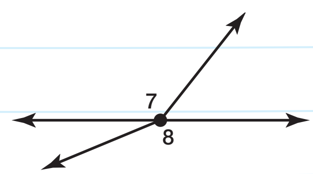
$\angle 5$ and $\angle 6$ are *not* adjacent angles.



$\angle 3$ and $\angle 4$ are adjacent angles.



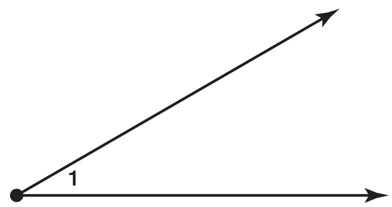
$\angle 7$ and $\angle 8$ are *not* adjacent angles.



1. Analyze the worked example. Then answer each question.

   a. Describe adjacent angles.

b. Draw ∠2 so that it is adjacent to ∠1.

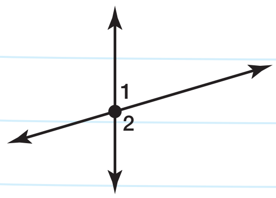


c. Is it possible to draw two angles that share a common vertex but do not share a common side? If so, draw an example. If not, explain why not.

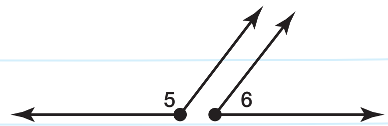
d. Is it possible to draw two angles that share a common side, but do not share a common vertex? If so, draw an example. If not, explain why not.

Adjacent angles are two angles that share a common vertex and share a common side.

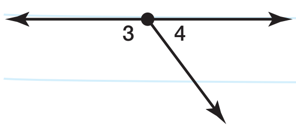
∠1 and ∠2 form a linear pair.



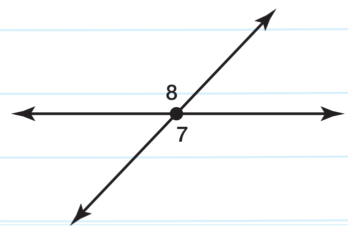
∠5 and ∠6 do *not* form a linear pair.



∠3 and ∠4 form a linear pair.



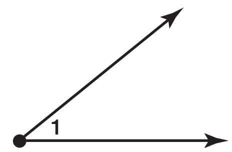
∠7 and ∠8 do *not* form a linear pair.



2. Analyze the worked example. Then answer each question.

a. Describe a linear pair of angles.

b. Draw ∠2 so that it forms a linear pair with ∠1.



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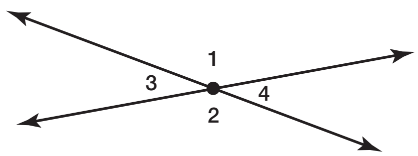
c. Name all linear pairs in the figure shown.



d. If the angles that form a linear pair are congruent, what can you conclude?

A **linear pair** of angles are two adjacent angles that have noncommon sides that form a line.

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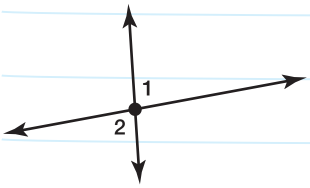
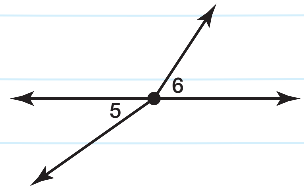
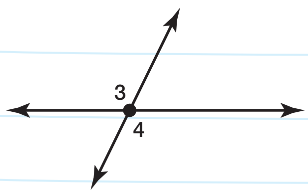
$\angle 1$ and $\angle 2$ are vertical angles.



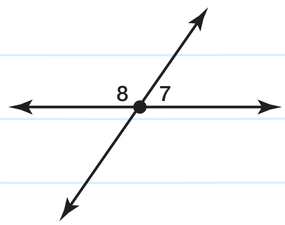
$\angle 5$ and $\angle 6$ are *not* vertical angles.



$\angle 3$ and $\angle 4$ are vertical angles.



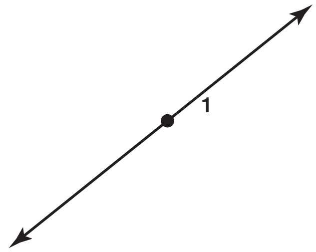
$\angle 7$ and $\angle 8$ are *not* vertical angles.



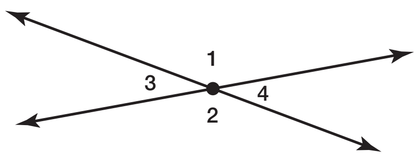
3. Analyze the worked example. Then answer each question.

a. Describe vertical angles.

b. Draw ∠2 so that it forms a vertical angle with ∠1.



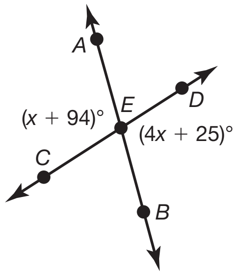
c. Name all vertical angle pairs in the diagram shown.

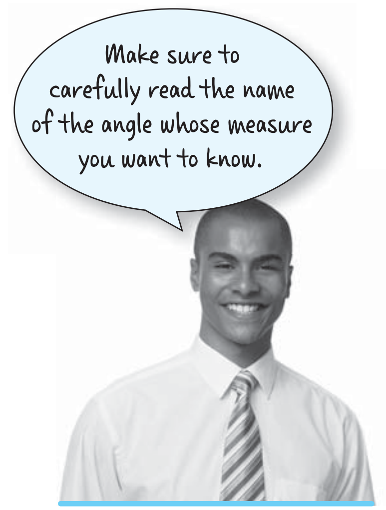


d. Measure each angle in part (c). What do you notice?

Vertical angles are two nonadjacent angles that are formed by two intersecting lines.

4. Determine $m\angle AED$. Explain how you determined the angle measure.





5. For each conditional statement, draw a diagram and then write the hypothesis as the "Given" and the conclusion as the "Prove."

a. $m\angle DEG + m\angle GEF = 180^\circ$, if $\angle DEG$ and $\angle GEF$ are a linear pair.

Given:

Prove:

b. If $\angle ABD$ and $\angle DBC$ are complementary, then $\overrightarrow{BA} \perp \overrightarrow{BC}$.

Given:

Prove:

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c. If $\angle 2$ and $\angle 3$ are vertical angles, then $\angle 2 \cong \angle 3$.

Given:

Prove:

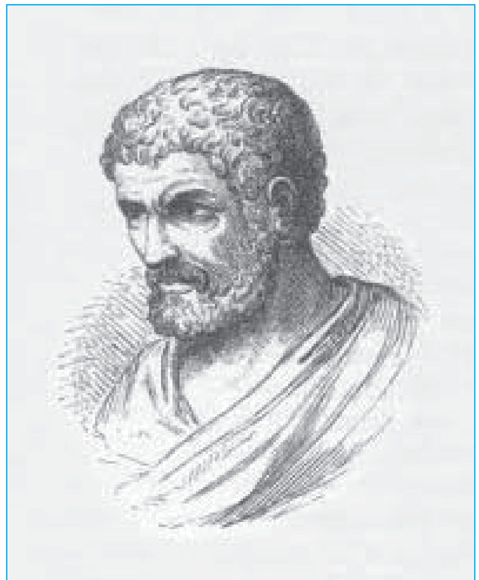
PROBLEM 3 Postulates and Theorems

A **postulate** is a statement that is accepted without proof.

A **theorem** is a statement that can be proven.

2

The Elements is a book written by the Greek mathematician Euclid. He used a small number of undefined terms and postulates to systematically prove many theorems. As a result, Euclid was able to develop a complete system we now know as **Euclidean geometry**.



Greek mathematician Euclid is sometimes referred to as the Father of Geometry.

Euclid's first five postulates are:

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one endpoint as center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough. (This postulate is equivalent to what is known as the parallel postulate.)

Euclid used only the first four postulates to prove the first 28 propositions or theorems of *The Elements*, but was forced to use the fifth postulate, the parallel postulate, to prove the 29th theorem.

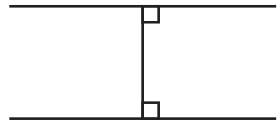
The *Elements* also includes five "common notions":

1. Things that equal the same thing also equal one another.
2. If equals are added to equals, then the wholes are equal.
3. If equals are subtracted from equals, then the remainders are equal.
4. Things that coincide with one another equal one another.
5. The whole is greater than the part.

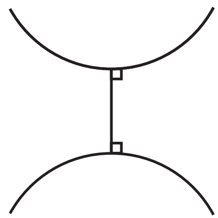
It is important to note that Euclidean geometry is not the only system of geometry. Examples of non-Euclidian geometries include hyperbolic and elliptic geometry. The essential difference between Euclidean and non-Euclidean geometry is the nature of parallel lines.

Another way to describe the differences between these geometries is to consider two lines in a plane that are both perpendicular to a third line.

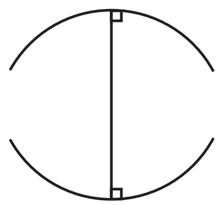
- In Euclidean geometry, the lines remain at a constant distance from each other and are known as parallels.



- In hyperbolic geometry, the lines "curve away" from each other.



- In elliptic geometry, the lines "curve toward" each other and eventually intersect.



Using this textbook as a guide, you will develop your own system of geometry, just like Euclid. You already used the three undefined terms *point, line,* and *plane* to define related terms such as *line segment* and *angle*.

Your journey continues with the introduction of three fundamental postulates:

- The Linear Pair Postulate
- The Segment Addition Postulate
- The Angle Addition Postulate

You will use these postulates to make various conjectures. If you are able to prove your conjectures, then the conjectures will become theorems. These theorems can then be used to make even more conjectures, which may also become theorems. Mathematicians use this process to create new mathematical ideas.

The **Linear Pair Postulate** states: "If two angles form a linear pair, then the angles are supplementary."

1. Use the Linear Pair Postulate to complete each representation.

   a. Sketch and label a linear pair.

   b. Use your sketch and the Linear Pair Postulate to write the hypothesis.

   c. Use your sketch and the Linear Pair Postulate to write the conclusion.

   d. Use your conclusion and the definition of supplementary angles to write a statement about the angles in your figure.

The **Segment Addition Postulate** states: "If point *B* is on $\overline{AC}$ and between points *A* and *C*, then $AB + BC = AC$."

2. Use the Segment Addition Postulate to complete each representation.

a. Sketch and label collinear points *D*, *E*, and *F* with point *E* between points *D* and *F*.

2

b. Use your sketch and the Segment Addition Postulate to write the hypothesis.

c. Use your sketch and the Segment Addition Postulate to write the conclusion.

d. Write your conclusion using measure notation.

The **Angle Addition Postulate** states: "If point D lies in the interior of $\angle ABC$, then $m\angle ABD + m\angle DBC = m\angle ABC$."

3. Use the Angle Addition Postulate to complete each representation.

a. Sketch and label $\angle DEF$ with $\overrightarrow{EG}$ drawn in the interior of $\angle DEF$.

2

b. Use your sketch and the Angle Addition Postulate to write the hypothesis.

c. Use your sketch and the Angle Addition Postulate to write the conclusion.

Be prepared to share your solutions and methods.

2.3

Forms of Proof

Paragraph Proof, Two-Column Proof, Construction Proof, and Flow Chart Proof

LEARNING GOALS

In this lesson, you will:

- Use the addition and subtraction properties of equality.
- Use the reflexive, substitution, and transitive properties.
- Write a paragraph proof.
- Prove theorems involving angles.
- Complete a two-column proof.
- Perform a construction proof.
- Complete a flow chart proof.

KEY TERMS

- Addition Property of Equality
- Subtraction Property of Equality
- Reflexive Property
- Substitution Property
- Transitive Property
- flow chart proof
- two-column proof
- paragraph proof
- construction proof
- Right Angle Congruence Theorem
- Congruent Supplement Theorem
- Congruent Complement Theorem
- Vertical Angle Theorem

Have you ever heard the famous phrase, "The proof is in the pudding"? If you stop to think about what this phrase means, you might be left scratching your head!

The phrase used now is actually a shortened version of the original phrase, "The proof of the pudding is in the eating." This phrase meant that the pudding recipe may appear to be delicious, include fresh ingredients, and may even look delicious after it is made. However, the only way to really "prove" that the pudding is delicious is by eating it!

Today it is used to imply that the quality or truth of something can only be determined by putting it into action. For example, you don't know how good an idea is until you actually test the idea.

Can you think of any other popular phrases that don't seem to make sense? Perhaps you should do a little research to find out where these phrases came from.

PROBLEM 1 Properties of Real Numbers in Geometry

2

Many properties of real numbers can be applied in geometry. These properties are important when making conjectures and proving new theorems.

The **Addition Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a + c = b + c$."

The Addition Property of Equality can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} + m\overline{EF} = m\overline{CD} + m\overline{EF}$.

Distances:

If $AB = CD$, then $AB + EF = CD + EF$.

1. Sketch a diagram and write a statement that applies the Addition Property of Equality to angle measures.

2. Sketch a diagram and write a statement that applies the Addition Property of Equality to segment measures.

The **Subtraction Property of Equality** states: "If a, b, and c are real numbers and $a = b$, then $a - c = b - c$."

The Subtraction Property of Equality can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = m\angle 2$, then $m\angle 1 - m\angle 3 = m\angle 2 - m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} - m\overline{EF} = m\overline{CD} - m\overline{EF}$.

Distances:

If $AB = CD$, then $AB - EF = CD - EF$.

2

3. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to angle measures.

4. Sketch a diagram and write a statement that applies the Subtraction Property of Equality to segment measures.

The **Reflexive Property** states: "If a is a real number, then $a = a$."

The Reflexive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

Angle measures:

$m\angle 1 = m\angle 1$

Segment measures:

$m\overline{AB} = m\overline{AB}$

Distances:

$AB = AB$

Congruent angles:

$\angle 1 \cong \angle 1$

Congruent segments:

$\overline{AB} \cong \overline{AB}$

5. Sketch a diagram and write a statement that applies the Reflexive Property to angles.

6. Sketch a diagram and write a statement that applies the Reflexive Property to segments.

The **Substitution Property** states: "If a and b are real numbers and $a = b$, then a can be substituted for b."

The Substitution Property can be applied to angle measures, segment measures, and distances.

Angle measures:

If $m\angle 1 = 56°$ and $m\angle 2 = 56°$, then $m\angle 1 = m\angle 2$.

Segment measures:

If $m\overline{AB} = 4$ mm and $m\overline{CD} = 4$ mm, then $m\overline{AB} = m\overline{CD}$.

Distances:

If $AB = 12$ ft and $CD = 12$ ft, then $AB = CD$.

2

7. Sketch a diagram and write a statement that applies the Substitution Property to angles.

8. Sketch a diagram and write a statement that applies the Substitution Property to segments.

The **Transitive Property** states: "If a, b, and c are real numbers, $a = b$, and $b = c$, then $a = c$."

The Transitive Property can be applied to angle measures, segment measures, distances, congruent angles, and congruent segments.

Angle measures:

If $m\angle 1 = m\angle 2$ and $m\angle 2 = m\angle 3$, then $m\angle 1 = m\angle 3$.

Segment measures:

If $m\overline{AB} = m\overline{CD}$ and $m\overline{CD} = m\overline{EF}$, then $m\overline{AB} = m\overline{EF}$.

Distances:

If $AB = CD$ and $CD = EF$, then $AB = EF$.

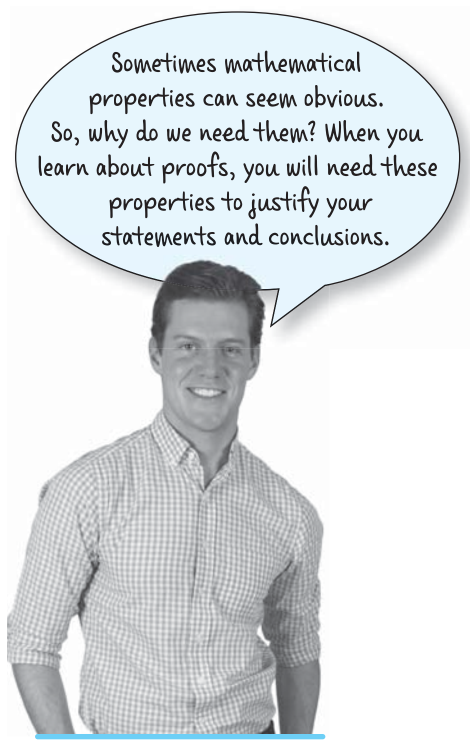
Congruent angles:

If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.

Congruent segments:

If $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, then $\overline{AB} \cong \overline{EF}$.

9. Sketch a diagram and write a statement that applies the Transitive Property to angles.



10. Sketch a diagram and write a statement that applies the Transitive Property to congruent segments.

PROBLEM 2 Various Forms of Proof

A **proof** is a logical series of statements and corresponding reasons that starts with a hypothesis and arrives at a conclusion. In this course, you will use four different kinds of proof.

1. The diagram shows four collinear points A, B, C, and D such that point B lies between points A and C, point C lies between points B and D, and $\overline{AB} \cong \overline{CD}$.

Consider the conditional statement: If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

a. Write the hypothesis as the "Given" and the conclusion as the "Prove."

Given:

Prove:

A **flow chart proof** is a proof in which the steps and reasons for each step are written in boxes. Arrows connect the boxes and indicate how each step and reason is generated from one or more other steps and reasons.

b. Cut out the steps on the flow chart proof.

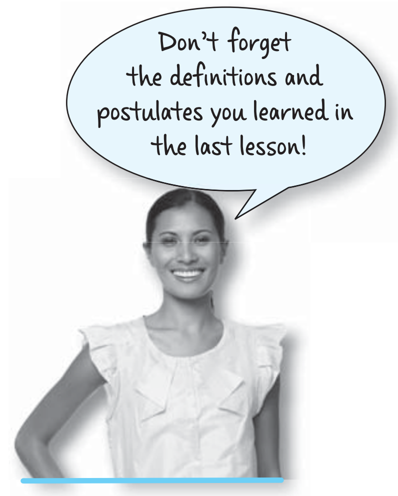
$\overline{AB} \cong \overline{CD}$ Given	$\overline{AC} \cong \overline{BD}$ Definition of congruent segments
$m\overline{AB} = m\overline{CD}$ Definition of congruent segments	$m\overline{BC} = m\overline{BC}$ Reflexive Property
$m\overline{AB} + m\overline{BC} = m\overline{CD} + m\overline{BC}$ Addition Property of Equality	$m\overline{AB} + m\overline{BC} = m\overline{AC}$ Segment Addition Postulate
$m\overline{AC} = m\overline{BD}$ Substitution Property	$m\overline{BC} + m\overline{CD} = m\overline{BD}$ Segment Addition Postulate

2

c. Complete the flow chart proof of the conditional statement in Question 1 by assembling your cutout steps in order. Use arrows to show the order of the flow chart proof.

Given:

Prove:



A **two-column proof** is a proof in which the steps are written in the left column and the corresponding reasons are written in the right column. Each step and corresponding reason are numbered.

d. Create a two-column proof of the conditional statement in Question 1.
Each box of the flow chart proof in Question 1, part (c) should appear as a row in the two-column proof.

Given:

Prove:

Statements	Reasons

A **paragraph proof** is a proof in which the steps and corresponding reasons are written in complete sentences.

e. Write a paragraph proof of the conditional statement in Question 1. Each row of the two-column proof in Question 1, part (d) should appear as a sentence in the paragraph proof.

A **construction proof** is a proof that results from creating an object with specific properties using only a compass and a straightedge.

f. Create a proof by construction of the conditional statement in Question 2.

Given:

Prove:

PROBLEM 3 Proof of the Right Angle Congruence Theorem

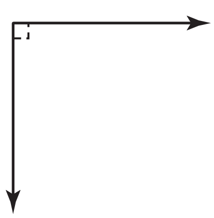


Fig. 1

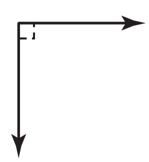
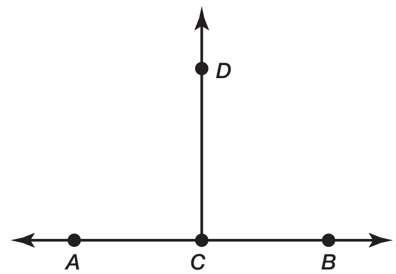


Fig. 2

1. Karl insists the angle in Figure 1 is larger than the angle in Figure 2. Roy disagrees and insists both angles are the same size. Who is correct? What is your reasoning?

2

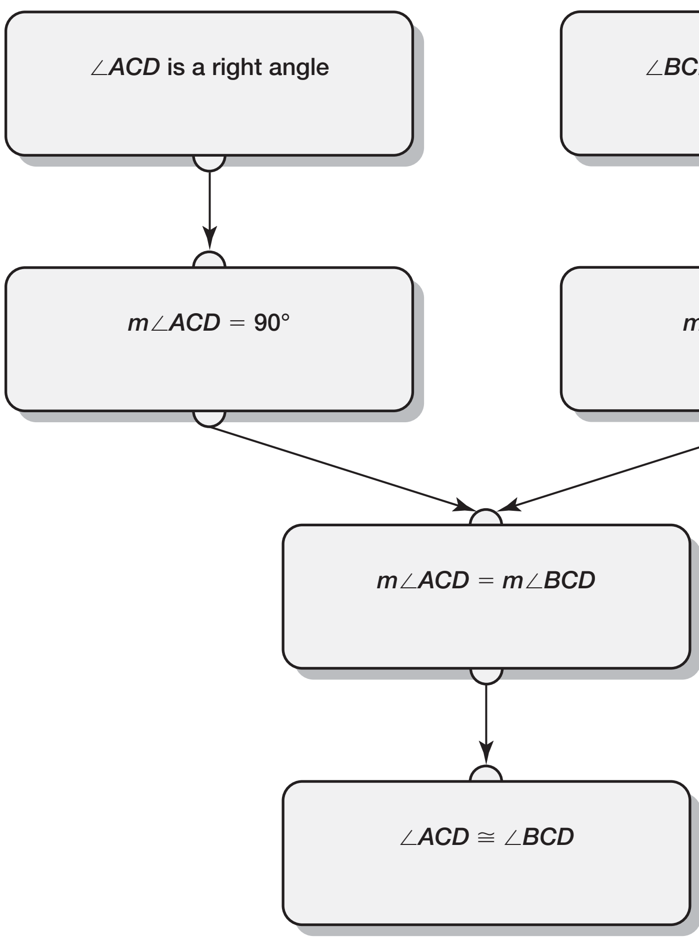
The **Right Angle Congruence Theorem** states: "All right angles are congruent."



Given: $\angle ACD$ and $\angle BCD$ are right angles.

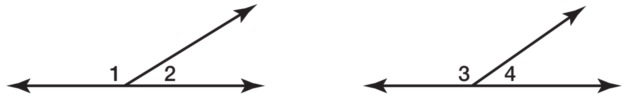
Prove: $\angle ACD \cong \angle BCD$

Complete the flow chart of the Right Angle Congruence Theorem by writing the statement for each reason in the boxes provided.



PROBLEM 4 Proofs of the Congruent Supplement Theorem

The **Congruent Supplement Theorem** states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent."



1. Use the diagram to write the "Given" statements for the Congruent Supplement Theorem. The "Prove" statement is provided.

Given:

Given:

Given:

Prove: $\angle 1 \cong \angle 3$

2. Cut out the steps of the flow chart proof.

$\angle 1$ is supplementary to $\angle 2$
Given

$\angle 3$ is supplementary to $\angle 4$
Given

$m\angle 1 + m\angle 2 = 180°$
Definition of supplementary angles

$\angle 2 \cong \angle 4$
Given

$m\angle 3 + m\angle 4 = 180°$
Definition of supplementary angles

$m\angle 1 = m\angle 3$
Subtraction Property of Equality

$m\angle 2 = m\angle 4$
Definition of congruent angles

$m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$
Substitution Property

$\angle 1 \cong \angle 3$
Definition of congruent angles

2

3. Complete the flow chart proof of the Congruent Supplements Theorem by assembling your cutout steps in order. Use arrows to show the order of the flow chart proof.

2

4. Create a two-column proof of the Congruent Supplement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

Statements	Reasons

2

PROBLEM 5 Proofs of the Congruent Complement Theorem

The **Congruent Complement Theorem** states: "If two angles are complements of the same angle or of congruent angles, then they are congruent."

1. Draw and label a diagram illustrating this theorem.

2. Use your diagram to write the "Given" and "Prove" statements for the Congruent Complement Theorem.

   Given:

   Given:

   Given:

   Prove:

3. Create a flow chart proof of the Congruent Complement Theorem.

2

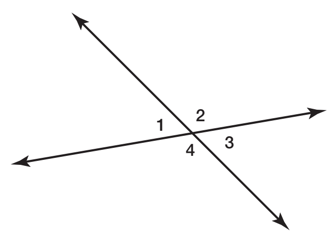
4. Create a two-column proof of the Congruent Complement Theorem. Each box of the flow chart proof in Question 3 should appear as a row in the two-column proof.

Statements	Reasons

2

PROBLEM 6 Proofs of the Vertical Angle Theorem

1. The **Vertical Angle Theorem** states: "Vertical angles are congruent."



2. Use the diagram to write the "Prove" statements for the Vertical Angle Theorem. The "Given" statements are provided.

   Given: $\angle 1$ and $\angle 2$ are a linear pair

   Given: $\angle 2$ and $\angle 3$ are a linear pair

   Given: $\angle 3$ and $\angle 4$ are a linear pair

   Given: $\angle 4$ and $\angle 1$ are a linear pair

   Prove:

   Prove:

3. Create a flow chart proof of the first “Prove” statement of the Vertical Angle Theorem.

2

4. Create a two-column proof of the second "Prove" statement of the Vertical Angle Theorem.

Given:

Given:

Prove:

Statements	Reasons

2

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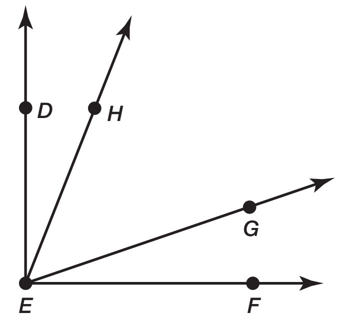
PROBLEM 7 Proofs Using the Angle Addition Postulate

Given: $\angle DEG \cong \angle HEF$
Prove: $\angle DEH \cong \angle GEF$



1. Prove the conditional statement using any method you choose.

Talk the Talk

1. List the advantages and disadvantages of each form of proof.

   a. flow chart proof

   b. two-column proof

   c. paragraph proof

   d. construction proof

2. Which form of proof do you prefer? Explain.

Once a theorem has been proven, it can be used as a reason in another proof. Using theorems that have already been proven allows you to write shorter proofs.

In this chapter, you proved these theorems:

- The Right Angle Congruence Theorem: All right angles are congruent.
- The Congruent Supplement Theorem: Supplements of congruent angles, or of the same angle, are congruent.
- The Congruent Complement Theorem: Complements of congruent angles, or of the same angle, are congruent.
- The Vertical Angle Theorem: Vertical angles are congruent.

A list of theorems that you prove throughout this course will be an excellent resource as you continue to make new conjectures and expand your system of geometry.

Be prepared to share your solutions and methods.

2.4

What's Your Proof?

Angle Postulates and Theorems

LEARNING GOALS

In this lesson, you will:

- Use the Corresponding Angle Postulate.
- Prove the Alternate Interior Angle Theorem.
- Prove the Alternate Exterior Angle Theorem.
- Prove the Same-Side Interior Angle Theorem.
- Prove the Same-Side Exterior Angle Theorem.

KEY TERMS

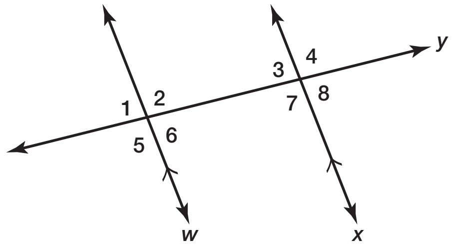
- Corresponding Angle Postulate
- conjecture
- Alternate Interior Angle Theorem
- Alternate Exterior Angle Theorem
- Same-Side Interior Angle Theorem
- Same-Side Exterior Angle Theorem

You are constantly bombarded with information through magazines, newspapers, television, and the Internet. However, not all "facts" that you read about are actually true! If you want to be an educated consumer of information, you should always be looking for the argument, or proof, to back up a statement. If you can't find such information then you should be skeptical.

Sometimes you need to carefully examine the evidence. For example, say someone claims that 4 out of 5 dentists recommend a certain toothpaste. Sounds pretty impressive, right? However, what if you learned that only five dentists were asked their opinions? You might start to question the claim. What if you also learned that the dentists were paid by the toothpaste company for their opinions? As you can see, sometimes the "truth" isn't always what it appears to be.

PROBLEM 1 The Corresponding Angle Postulate

The **Corresponding Angle Postulate** states: "If two parallel lines are intersected by a transversal, then corresponding angles are congruent."



1. Name all pairs of angles that are congruent using the Corresponding Angle Postulate.

A **conjecture** is a hypothesis that something is true. The hypothesis can later be proved or disproved.

2. Write a conjecture about each pair of angles formed by parallel lines cut by a transversal. Explain how you made each conjecture.

   a. alternate interior angles.

   b. alternate exterior angles.

2

c. same-side interior angles

2

d. same-side exterior angles

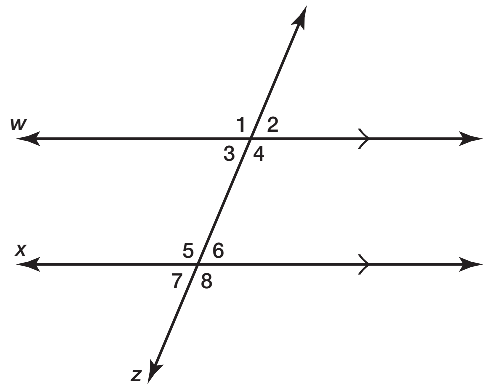
3. Did you use inductive or deductive reasoning to make each conjecture?

PROBLEM 2 Conjecture or Theorem?

2

If you can prove that a conjecture is true, then it becomes a theorem.

1. The Alternate Interior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then alternate interior angles are congruent."

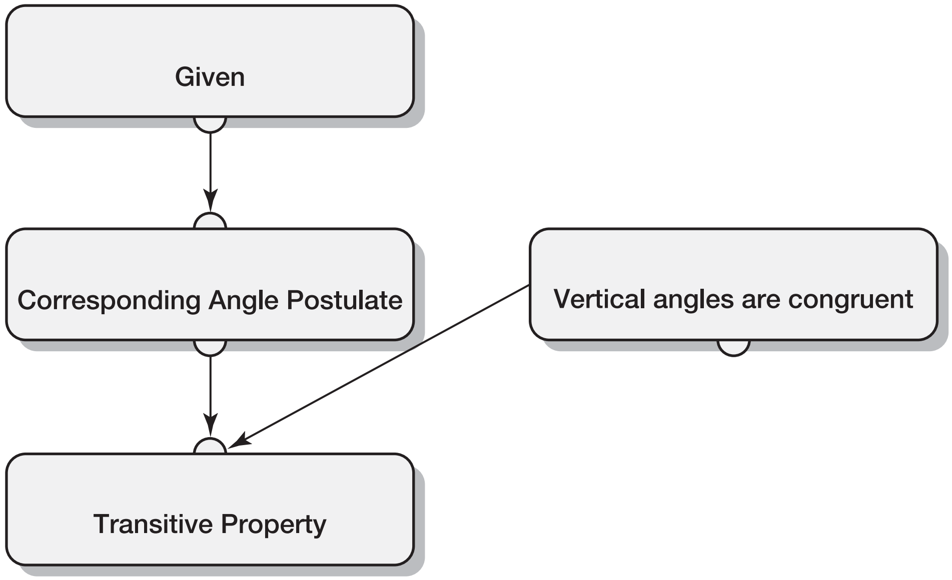


a. Use the diagram to write the "Given" and "Prove" statements for the Alternate Interior Angle Conjecture.

Given:

Prove:

b. Complete the flow chart proof of the Alternate Interior Angle Conjecture by writing the reason for each statement in the boxes provided.



c. Create a two-column proof of the Alternate Interior Angle Theorem.

Statements	Reasons

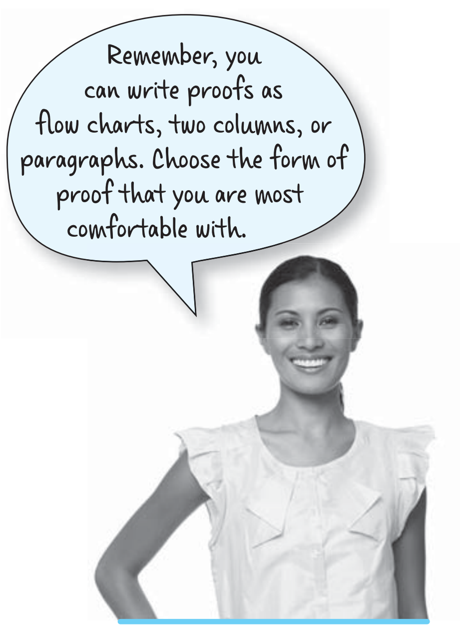
You have just proven the Alternate Interior Angle Conjecture. It is now known as the **Alternate Interior Angle Theorem**.

2. The Alternate Exterior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent."

a. Draw and label a diagram illustrating the Alternate Exterior Angle Conjecture. Then, write the given and prove statements.

2

b. Prove the Alternate Exterior Angle Conjecture.

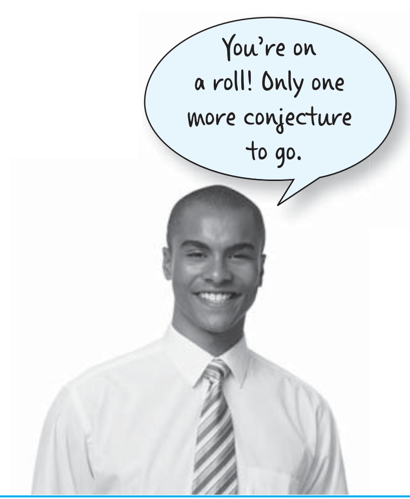


You have just proven the Alternate Exterior Angle Conjecture. It is now known as the **Alternate Exterior Angle Theorem**.

3. The Same-Side Interior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are supplementary."

   a. Draw and label a diagram illustrating the Same-Side Interior Angle Conjecture. Then, write the given and prove statements.

   b. Prove the Same-Side Interior Angle Conjecture.



You have just proven the Same-Side Interior Angle Conjecture. It is now known as the **Same-Side Interior Angle Theorem**.

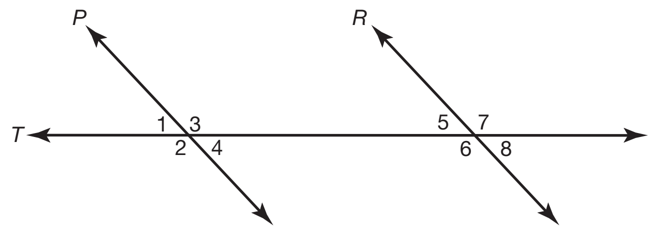
4. The Same-Side Exterior Angle Conjecture states: "If two parallel lines are intersected by a transversal, then exterior angles on the same side of the transversal are supplementary."

   a. Draw and label a diagram illustrating the Same-Side Exterior Angle Conjecture. Then, write the given and prove statements.

   b. Prove the Same-Side Exterior Angle Conjecture.

You have just proven the Same-Side Exterior Angle Conjecture. It is now known as the **Same-Side Exterior Angle Theorem**.

Talk the Talk



Given: $m\angle 4 = 37°$

1. Gail determined the measures of all eight angles labeled using the given information. Stu said she could only calculate the measure of four angles with certainty. Who is correct? Explain your reasoning.

If two parallel lines are intersected by a transversal, then:

- corresponding angles are congruent.
- alternate interior angles are congruent.
- alternate exterior angles are congruent.
- same-side interior angles are supplementary.
- same-side exterior angles are supplementary.

2

Each of these relationships is represented by a postulate or theorem.

- **Corresponding Angle Postulate**: If two parallel lines are intersected by a transversal, then corresponding angles are congruent.
- **Alternate Interior Angle Theorem**: If two parallel lines are intersected by a transversal, then alternate interior angles are congruent.
- **Alternate Exterior Angle Theorem**: If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent.
- **Same-Side Interior Angle Theorem**: If two parallel lines are intersected by a transversal, then interior angles on the same side of the transversal are supplementary.
- **Same-Side Exterior Angle Theorem**: If two parallel lines are intersected by a transversal, then exterior angles on the same side of the transversal are supplementary.

2. Did you use inductive or deductive reasoning to prove each theorem?

Be prepared to share your solutions and methods.

2.5

A Reversed Condition

Parallel Line Converse Theorems

LEARNING GOALS

In this lesson, you will:

- Write and prove parallel line converse conjectures.

KEY TERMS

- converse
- Corresponding Angle Converse Postulate
- Alternate Interior Angle Converse Theorem
- Alternate Exterior Angle Converse Theorem
- Same-Side Interior Angle Converse Theorem
- Same-Side Exterior Angle Converse Theorem

Lewis Carroll is best known as the author of *Alice's Adventures in Wonderland* and its sequel *Through the Looking Glass*. However, Carroll also wrote several mathematics books, many of which focus on logic. In fact, Carroll included logic in many of his fiction books. Sometimes these took the form of "logical nonsense" such as the tea party scene with the Mad Hatter.

At one point of the scene, Alice proclaims that she says what she means, or at least, that she means what she says, insisting that the two statements are the same thing. The numerous attendees of the tea party then correct her with a series of flipped sentences which have totally different meanings. For example, "I like what I get" and "I get what I like".

Are these two sentences saying the same thing? Can you think of other examples of flipped sentences?

PROBLEM 1 Converses

The **converse** of a conditional statement written in the form "If *p*, then *q*" is the statement written in the form "If *q*, then *p*." The converse is a new statement that results when the hypothesis and conclusion of the conditional statement are interchanged.

The Corresponding Angle Postulate states: "If two parallel lines are intersected by a transversal, then the corresponding angles are congruent."

2

The **Corresponding Angle Converse Postulate** states: "If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel."

The Corresponding Angle Converse Postulate is used to prove new conjectures formed by writing the converses of the parallel lines theorems.

1. For each theorem:
   - Identify the hypothesis *p* and conclusion *q*.
   - Write the converse of the theorem as a conjecture.

   a. Alternate Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate interior angles are congruent.

   Hypothesis *p*:

   Conclusion *q*:

   Alternate Interior Angle Converse Conjecture:

   b. Alternate Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the alternate exterior angles are congruent.

   Hypothesis *p*:

   Conclusion *q*:

   Alternate Exterior Angle Converse Conjecture:

   c. Same-Side Interior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side interior angles are supplementary.

   Hypothesis *p*:

   Conclusion *q*:

   Same-Side Interior Angle Converse Conjecture:

d. Same-Side Exterior Angle Theorem: If two parallel lines are intersected by a transversal, then the same-side exterior angles are supplementary.

Hypothesis p:

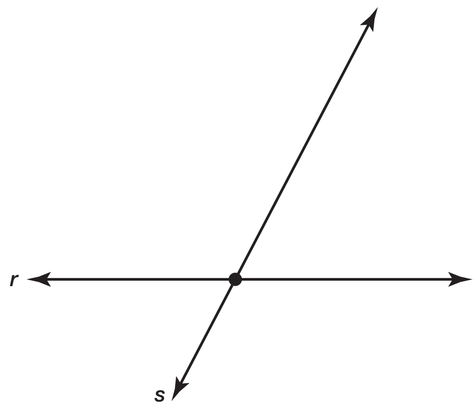
Conclusion q:

Same-Side Exterior Angle Converse Conjecture:

2

2. Consider lines r and s.

a. Use the Corresponding Angle Converse Postulate to construct a line parallel to line r. Write the steps.

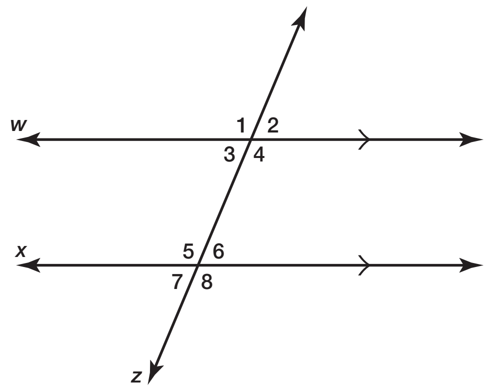


b. Which line is a transversal?

c. Which lines are parallel?

PROBLEM 2 Proving the Parallel Line Converse Conjectures

1. The Alternate Interior Angle Converse Conjecture states: "If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Alternate Interior Angle Converse Conjecture.

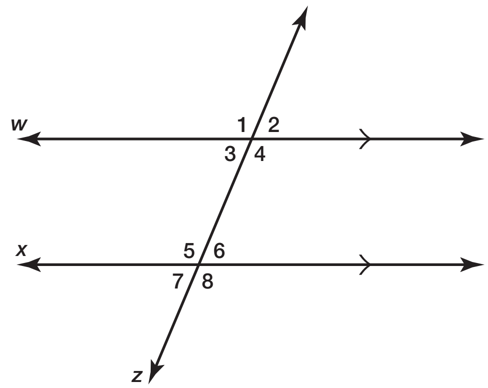
Given:

Prove:

b. Prove the Alternate Interior Angle Converse Conjecture.

You have just proven the Alternate Interior Angle Converse Conjecture. It is now known as the **Alternate Interior Angle Converse Theorem**.

2. The Alternate Exterior Angle Converse Conjecture states: "If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Alternate Exterior Angle Converse Conjecture.

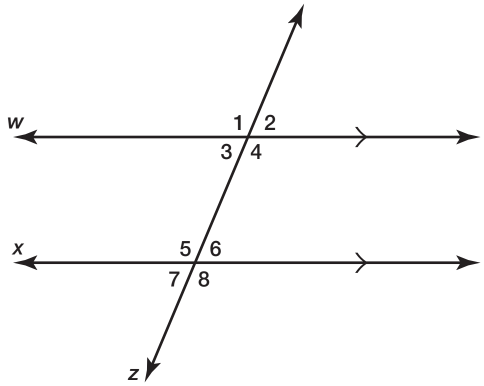
Given:

Prove:

b. Prove the Alternate Exterior Angle Converse Conjecture.

You have just proven the Alternate Exterior Angle Converse Conjecture. It is now known as the **Alternate Exterior Angle Converse Theorem**.

3. The Same-Side Interior Angle Converse Conjecture states: "If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Same-Side Interior Angle Converse Conjecture.

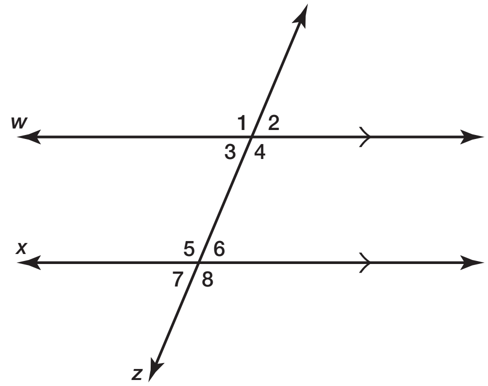
Given:

Prove:

b. Prove the Same-Side Interior Angle Converse Conjecture.

You have just proven the Same-Side Interior Angle Converse Conjecture. It is now known as the **Same-Side Interior Angle Converse Theorem**.

4. The Same-Side Exterior Angle Converse Conjecture states: "If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel."



a. Use the diagram to write the given and prove statements for the Same-Side Exterior Angle Converse Conjecture.

Given:

Prove:

b. Prove the Same-Side Exterior Angle Converse Conjecture.

2

You have just proven the Same-Side Exterior Angle Converse Conjecture. It is now known as the **Same-Side Exterior Angle Converse Theorem**.

Talk the Talk

2

Here are all the converse postulates you have proven. Each converse conjecture you have proven is a new theorem.

Corresponding Angle Converse Postulate: If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel.

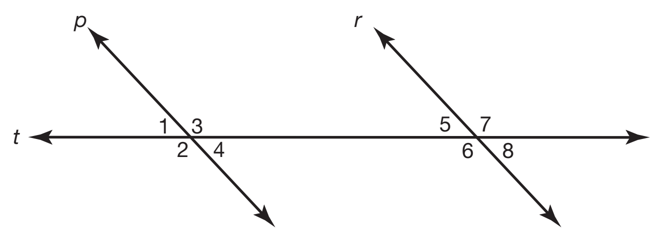
Alternate Interior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel.

Alternate Exterior Angle Converse Theorem: If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel.

Same-Side Interior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel.

Same-Side Exterior Angle Converse Theorem: If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel.

Use the diagram to answer the questions.



1. Which theorem or postulate would use $\angle 2 \cong \angle 7$ to justify line p is parallel to line r?

2. Which theorem or postulate would use $\angle 4 \cong \angle 5$ to justify line p is parallel to line r?

3. Which theorem or postulate would use $\angle 1 \cong \angle 5$ to justify line p is parallel to line r?

4. Which theorem or postulate would use $m\angle 4 + m\angle 6 = 180°$ to justify line p is parallel to line r?

5. Which theorem or postulate would use $m\angle 1 + m\angle 7 = 180°$ to justify line p is parallel to line r?

2

6. Which theorem or postulate would use line p is parallel to line r to justify $\angle 2 \cong \angle 7$?

7. Which theorem or postulate would use line p is parallel to line r to justify $\angle 4 \cong \angle 5$?

8. Which theorem or postulate would use line p is parallel to line r to justify $\angle 1 \cong \angle 5$?

9. Which theorem or postulate would use line p is parallel to line r to justify $m\angle 4 + m\angle 6 = 180°$?

10. Which theorem or postulate would use line p is parallel to line r to justify $m\angle 1 + m\angle 7 = 180°$?

Be prepared to share your methods and solutions.

2

Chapter 2 Summary

KEY TERMS

- induction (2.1)
- deduction (2.1)
- counterexample (2.1)
- conditional statement (2.1)
- propositional form (2.1)
- propositional variables (2.1)
- hypothesis (2.1)
- conclusion (2.1)
- truth value (2.1)
- truth table (2.1)
- supplementary angles (2.2)
- complementary angles (2.2)
- adjacent angles (2.2)
- linear pair (2.2)
- vertical angles (2.2)
- postulate (2.2)
- theorem (2.2)
- Euclidean geometry (2.2)
- Addition Property of Equality (2.3)
- Subtraction Property of Equality (2.3)
- Reflexive Property (2.3)
- Substitution Property (2.3)
- Transitive Property (2.3)
- flow chart proof (2.3)
- two-column proof (2.3)
- paragraph proof (2.3)
- construction proof (2.3)
- conjecture (2.4)
- converse (2.5)

POSTULATES AND THEOREMS

- Linear Pair Postulate (2.2)
- Segment Addition Postulate (2.2)
- Angle Addition Postulate (2.2)
- Right Angle Congruence Theorem (2.3)
- Congruent Supplement Theorem (2.3)
- Congruent Complement Theorem (2.3)
- Vertical Angle Theorem (2.3)
- Corresponding Angle Postulate (2.4)
- Alternate Interior Angle Theorem (2.4)
- Alternate Exterior Angle Theorem (2.4)
- Same-Side Interior Angle Theorem (2.4)
- Same-Side Exterior Angle Theorem (2.4)
- Corresponding Angle Converse Postulate (2.5)
- Alternate Interior Angle Converse Theorem (2.5)
- Alternate Exterior Angle Converse Theorem (2.5)
- Same-Side Interior Angle Converse
- Theorem (2.5)
- Same-Side Exterior Angle Converse Theorem (2.5)

2.1 Identifying and Comparing Induction and Deduction

Induction uses specific examples to make a conclusion. Induction, also known as inductive reasoning, is used when observing data, recognizing patterns, making generalizations about the observations or patterns, and reapplying those generalizations to unfamiliar situations. Deduction, also known as deductive reasoning, uses a general rule or premise to make a conclusion. It is the process of showing that certain statements follow logically from some proven facts or accepted rules.

2

Example

Kyra sees coins at the bottom of a fountain. She concludes that if she throws a coin into the fountain, it too will sink. Tyler understands the physical laws of gravity and mass and decides a coin he throws into the fountain will sink.

The specific information is the coins Kyra and Tyler observed at the bottom of the fountain.
The general information is the physical laws of gravity and mass.
Kyra's conclusion that her coin will sink when thrown into the fountain is induction.
Tyler's conclusion that his coin will sink when thrown into the fountain is deduction.

2.1 Identifying False Conclusions

It is important that all conclusions are tracked back to given truths. There are two reasons why a conclusion may be false. Either the assumed information is false or the argument is not valid.

Example

Erin noticed that every time she missed the bus, it rained. So, she concludes that next time she misses the bus it will rain.

Erin's conclusion is false because missing the bus is not related to what makes it rain.

2.1 Writing a Conditional Statement

A conditional statement is a statement that can be written in the form "If p, then q." The portion of the statement represented by p is the hypothesis. The portion of the statement represented by q is the conclusion.

Example

If I plant an acorn, then an oak tree will grow.

A solid line is drawn under the hypothesis, and a dotted line is drawn under the conclusion.

2.1 Using a Truth Table to Explore the Truth Value of a Conditional Statement

The truth value of a conditional statement is whether the statement is true or false. If a conditional statement could be true, then its truth value is considered "true." The first two columns of a truth table represent the possible truth values for *p* (the hypothesis) and *q* (the conclusion). The last column represents the truth value of the conditional statement ($p \to q$). Notice that the truth value of a conditional statement is either "true" or "false," but not both.

2

Example

Consider the conditional statement, "If I eat too much, then I will get a stomach ache."

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

When *p* is true, I ate too much. When *q* is true, I will get a stomach ache. It is true that when I eat too much, I will get a stomach ache. So, the truth value of the conditional statement is true.

When *p* is true, I ate too much. When *q* is false, I will not get a stomach ache. It is false that when I eat too much, I will not get a stomach ache. So, the truth value of the conditional statement is false.

When *p* is false, I did not eat too much. When *q* is true, I will get a stomach ache. It could be true that when I did not eat too much, I will get a stomach ache for a different reason. So, the truth value of the conditional statement in this case is true.

When *p* is false, I did not eat too much. When *q* is false, I will not get a stomach ache. It could be true that when I did not eat too much, I will not get a stomach ache. So, the truth value of the conditional statement in this case is true.

2.1 Rewriting Conditional Statements

A conditional statement is a statement that can be written in the form "If *p*, then *q*." The hypothesis of a conditional statement is the variable *p*. The conclusion of a conditional statement is the variable *q*.

Example

Consider the following statement: If two angles form a linear pair, then the sum of the measures of the angles is 180 degrees. The statement is a conditional statement. The hypothesis is "two angles form a linear pair," and the conclusion is "the sum of the measures of the angles is 180 degrees." The conditional statement can be rewritten with the hypothesis as the "Given" statement and the conclusion as the "Prove" statement.

Given: Two angles form a linear pair.

Prove: The sum of the measures of the angles is 180 degrees.

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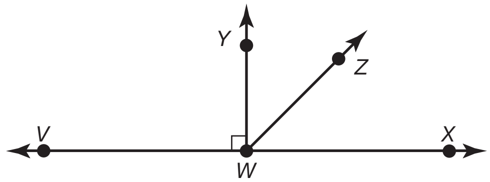
2.2 Identifying Complementary and Supplementary Angles

Two angles are supplementary if the sum of their measures is 180 degrees.
Two angles are complementary if the sum of their measures is 90 degrees.



Example

In the diagram above, angles *YWZ* and *ZWX* are complementary angles.

In the diagram above, angles *VWY* and *XWY* are supplementary angles.
Also, angles *VWZ* and *XWZ* are supplementary angles.

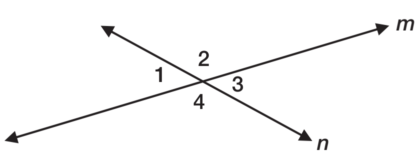
2.2 Identifying Adjacent Angles, Linear Pairs, and Vertical Angles

Adjacent angles are angles that share a common vertex and a common side.

A linear pair of angles consists of two adjacent angles that have noncommon sides that form a line.

Vertical angles are nonadjacent angles formed by two intersecting lines.

Example



Angles 2 and 3 are adjacent angles.

Angles 1 and 2 form a linear pair. Angles 2 and 3 form a linear pair. Angles 3 and 4 form a linear pair. Angles 4 and 1 form a linear pair.

Angles 1 and 3 are vertical angles. Angles 2 and 4 are vertical angles.

2.2 Determining the Difference Between Euclidean and Non-Euclidean Geometry

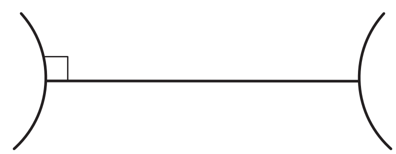
Euclidean geometry is a system of geometry developed by the Greek mathematician Euclid that included the following five postulates.

1. A straight line segment can be drawn joining any two points.
2. Any straight line segment can be extended indefinitely in a straight line.
3. Given any straight line segment, a circle can be drawn that has the segment as its radius and one point as the center.
4. All right angles are congruent.
5. If two lines are drawn that intersect a third line in such a way that the sum of the inner angles on one side is less than two right angles, then the two lines inevitably must intersect each other on that side if extended far enough.

Example

Euclidean geometry:

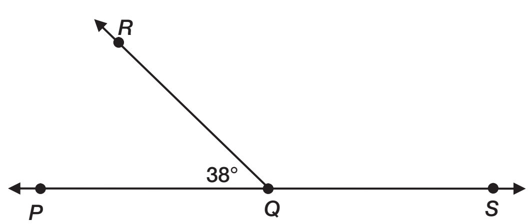
Non-Euclidean geometry:



2.2 Using the Linear Pair Postulate

The Linear Pair Postulate states: "If two angles form a linear pair, then the angles are supplementary."

Example



$m\angle PQR + m\angle SQR = 180°$

$38° + m\angle SQR = 180°$

$m\angle SQR = 180° - 38°$

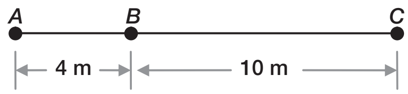
$m\angle SQR = 142°$

2

2.2 Using the Segment Addition Postulate

The Segment Addition Postulate states: "If point B is on segment AC and between points A and C, then $AB + BC = AC$."

Example



$AB + BC = AC$

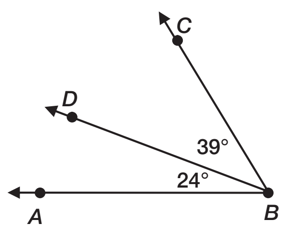
$4 \text{ m} + 10 \text{ m} = AC$

$AC = 14 \text{ m}$

2.2 Using the Angle Addition Postulate

The Angle Addition Postulate states: "If point D lies in the interior of angle ABC, then $m\angle ABD + m\angle DBC = m\angle ABC$."

Example



$m\angle ABD + m\angle DBC = m\angle ABC$

$24° + 39° = m\angle ABC$

$m\angle ABC = 63°$

2.3 Using Properties of Real Numbers in Geometry

The Addition Property of Equality states: "If a, b, and c are real numbers and $a = b$, then $a + c = b + c$."

The Subtraction Property of Equality states: "If a, b, and c are real numbers and $a = b$, then $a - c = b - c$."

The Reflexive Property states: "If a is a real number, then $a = a$."

The Substitution Property states: "If a and b are real numbers and $a = b$, then a can be substituted for b.

The Transitive Property states: "If a, b, and c are real numbers and $a = b$ and $b = c$, then $a = c$."

Example

Addition Property of Equality applied to angle measures: If $m\angle 1 = m\angle 2$, then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 3$.

Subtraction Property of Equality applied to segment measures: If $m\overline{AB} = m\overline{CD}$, then $m\overline{AB} - m\overline{EF} = m\overline{CD} - m\overline{EF}$.

Reflexive Property applied to distances: $AB = AB$

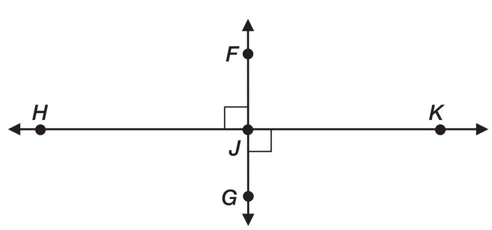
Substitution Property applied to angle measures: If $m\angle 1 = 20°$ and $m\angle 2 = 20°$, then $m\angle 1 = m\angle 2$.

Transitive Property applied to segment measures: If $m\overline{AB} = m\overline{CD}$ and $m\overline{CD} = m\overline{EF}$, then $m\overline{AB} = m\overline{EF}$.

2.3 Using the Right Angle Congruence Theorem

The Right Angle Congruence Theorem states: "All right angles are congruent."

Example



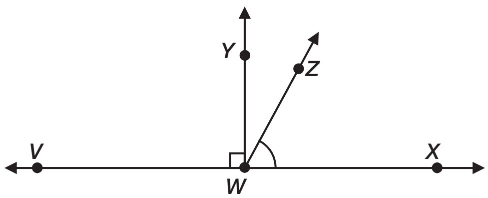
$\angle FJH \cong \angle GJK$

2

2.3 Using the Congruent Supplement Theorem

The Congruent Supplement Theorem states: "If two angles are supplements of the same angle or of congruent angles, then the angles are congruent."

Example

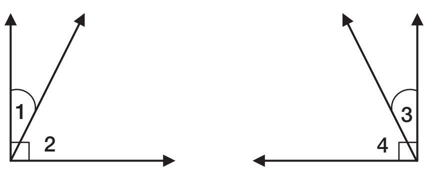


$\angle VWZ \cong \angle XWY$

2.3 Using the Congruent Complement Theorem

The Congruent Complement Theorem states: "If two angles are complements of the same angle or of congruent angles, then the angles are congruent."

Example

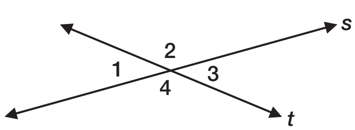


$\angle 2 \cong \angle 4$

2.3 Using the Vertical Angle Theorem

The Vertical Angle Theorem states: "Vertical angles are congruent."

Example

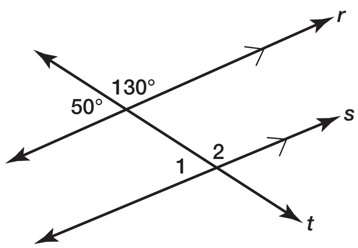


$\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$

2.4 Using the Corresponding Angle Postulate

The Corresponding Angle Postulate states: "If two parallel lines are intersected by a transversal, then corresponding angles are congruent."

Example



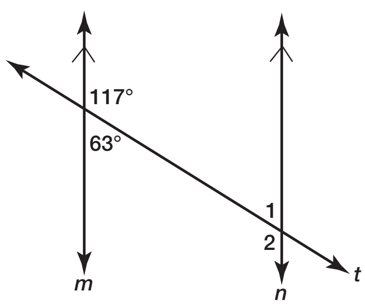
The angle that measures 50° and ∠1 are corresponding angles.
So, $m\angle 1 = 50°$.

The angle that measures 130° and ∠2 are corresponding angles.
So, $m\angle 2 = 130°$.

2.4 Using the Alternate Interior Angle Theorem

The Alternate Interior Angle Theorem states: "If two parallel lines are intersected by a transversal, then alternate interior angles are congruent."

Example



The angle that measures 63° and ∠1 are alternate interior angles.
So, $m\angle 1 = 63°$.

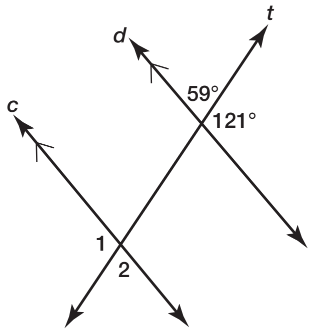
The angle that measures 117° and ∠2 are alternate interior angles.
So, $m\angle 2 = 117°$.

2.4

Using the Alternate Exterior Angle Theorem

The Alternate Exterior Angle Theorem states: "If two parallel lines are intersected by a transversal, then alternate exterior angles are congruent."

Example



The angle that measures 121° and $\angle 1$ are alternate exterior angles. So, $m\angle 1 = 121°$.

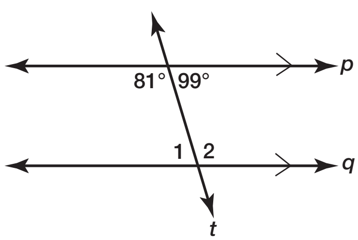
The angle that measures 59° and $\angle 2$ are alternate exterior angles. So, $m\angle 2 = 59°$.

2.4

Using the Same-Side Interior Angle Theorem

The Same-Side Interior Angle Theorem states: "If two parallel lines are intersected by a transversal, then same-side interior angles are supplementary."

Example



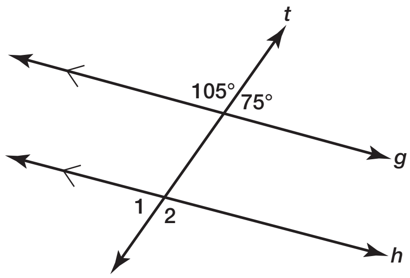
The angle that measures 81° and $\angle 1$ are same-side interior angles.
So, $m\angle 1 = 180° - 81° = 99°$.

The angle that measures 99° and $\angle 2$ are same-side interior angles.
So, $m\angle 2 = 180° - 99° = 81°$.

2.4 Using the Same-Side Exterior Angle Theorem

The Same-Side Exterior Angle Theorem states: “If two parallel lines are intersected by a transversal, then same-side exterior angles are supplementary.”

Example



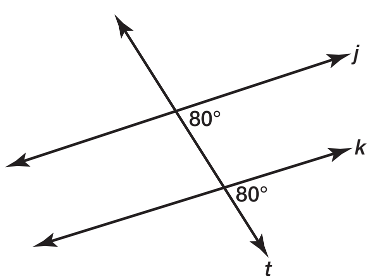
The angle that measures 105° and $\angle 1$ are same-side exterior angles.
So, $m\angle 1 = 180° - 105° = 75°$.

The angle that measures 75° and $\angle 2$ are same-side exterior angles.
So, $m\angle 2 = 180° - 75° = 105°$.

2.5 Using the Corresponding Angle Converse Postulate

The Corresponding Angle Converse Postulate states: “If two lines intersected by a transversal form congruent corresponding angles, then the lines are parallel.”

Example



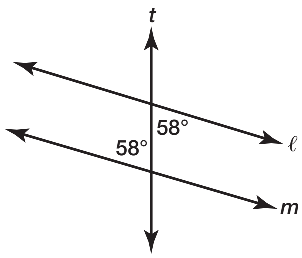
Corresponding angles have the same measure. So, $j \parallel k$.

2

2.5 Using the Alternate Interior Angle Converse Theorem

The Alternate Interior Angle Converse Theorem states: "If two lines intersected by a transversal form congruent alternate interior angles, then the lines are parallel."

Example

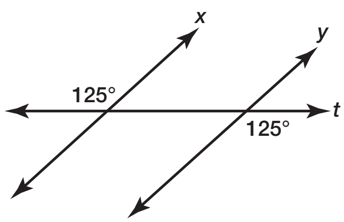


Alternate interior angles have the same measure. So, $\ell \parallel m$.

2.5 Using the Alternate Exterior Angle Converse Theorem

The Alternate Exterior Angle Converse Theorem states: "If two lines intersected by a transversal form congruent alternate exterior angles, then the lines are parallel."

Example

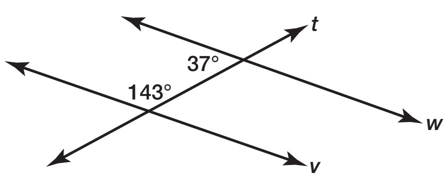


Alternate exterior angles have the same measure. So, $x \parallel y$.

2.5 Using the Same-Side Interior Angle Converse Theorem

The Same-Side Interior Angle Converse Theorem states: "If two lines intersected by a transversal form supplementary same-side interior angles, then the lines are parallel."

Example

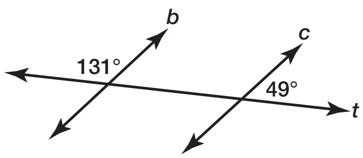


Same-side interior angles are supplementary: $37° + 143° = 180°$. So, $v \parallel w$.

2.5 Using the Same-Side Exterior Angle Converse Theorem

The Same-Side Exterior Angle Converse Theorem states: "If two lines intersected by a transversal form supplementary same-side exterior angles, then the lines are parallel."

Example



Same-side exterior angles are supplementary: $131° + 49° = 180°$. So, $b \parallel c$.

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