Perimeter and Area of Geometric Figures on the Coordinate Plane

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3.1

Transforming to a New Level!

Using Transformations to Determine Area

LEARNING GOALS

In this lesson, you will:

- Determine the areas of squares on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine the areas of rectangles using transformations.

You've probably been in a restaurant or another public building and seen a sign like this:

MAXIMUM OCCUPANCY **480**

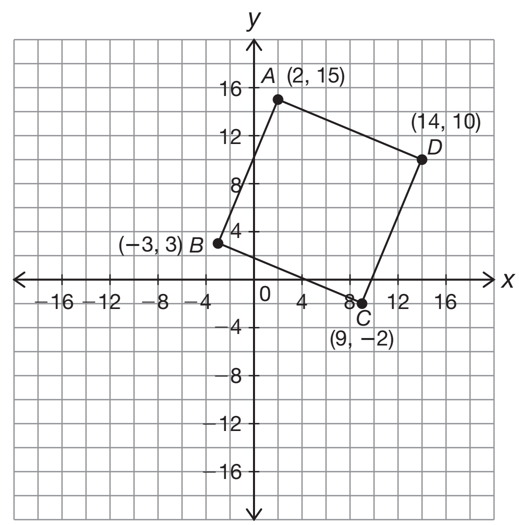
What does this mean? It means that the maximum number of people that can be in that space cannot—by law—be more than 480.

Why does this matter? Well, occupancy laws are often determined by the fire marshal of a town or city. If too many people are in a space when an emergency occurs, then getting out would be extremely difficult or impossible with everyone rushing for the exits. So, occupancy laws are there to protect people in case of emergencies like fires.

Of course, the area of a space is considered when determining maximum occupancy. Can you think of other factors that should be considered?

PROBLEM 1 Pomp and Circumstance

Marissa is throwing a party for her graduation and wants to invite all of her friends and their families. Consider the space defined by quadrilateral *ABCD*. Each of the four corners of the space is labeled with coordinates, measured in feet, and defines the dimensions of the room that Marissa's little brother says the party should be held.



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1. Marissa's mom says that the room is obviously a square or a rectangle, so if you can figure out the length of one or two of the sides, then you can easily determine the area.

Marissa tells her mother that you can't just assume that a shape is a square or a rectangle because it looks like one.

Who is correct and why?

2. List the properties of each shape.

   a. squares

   b. non-square rectangles

3. How can you use the properties you listed in Question 2 to determine whether the room is a square or a non-square rectangle?

4. A rule of thumb for determining the maximum occupancy of a room is that each person in the room is given 36 square feet of space.

   Predict the maximum occupancy of the room Marissa wants to rent. Describe the information you need and the strategies you could use to improve your prediction.

5. Determine if quadrilateral *ABCD* is a square or a rectangle. Show your work.

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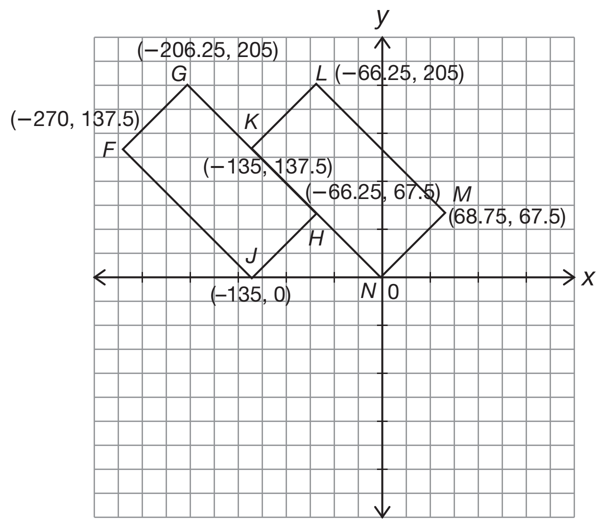
6. Use the rule of thumb from Question 4 to determine the maximum number of people that Marissa can invite to her party.

7. Do you think this location is reasonable for Marissa's graduation party? Why or why not?

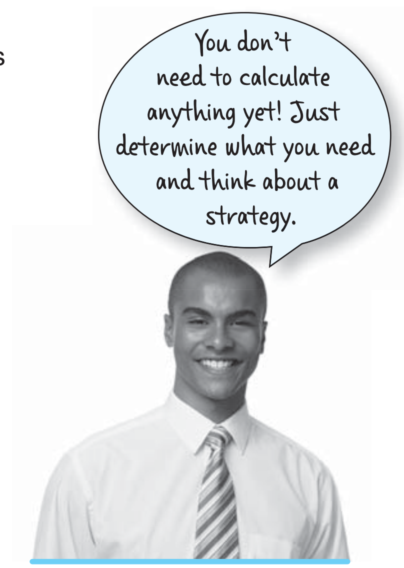
PROBLEM 2 Maximum Efficiency

In addition to formulas and properties, you can also use transformations to help make your problem solving more efficient.

The figure shown on the coordinate plane is composed of two quadrilaterals, *FGHJ* and *KLMN*.



1. Describe the information you need and the strategies you can use to determine the total area of the figure.



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2. Colby says that the two quadrilaterals are congruent. He says that knowing this can help him determine the area of the figure more efficiently. Is Colby correct? Explain your reasoning.

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3. Describe a transformation you can use to determine whether the two quadrilaterals are congruent. Explain why this transformation can prove congruency.

4. Apply the transformation you described in Question 3 to determine if the quadrilaterals are congruent. Show your work and explain your reasoning.

5. Tomas had an idea for solving the problem even more efficiently.

Tomas

When a polygon has vertices that are on the x- or y-axis or are at the origin, it is a little easier to use the Distance Formula, because one or more of the coordinates are 0.

Explain why Tomas is correct.

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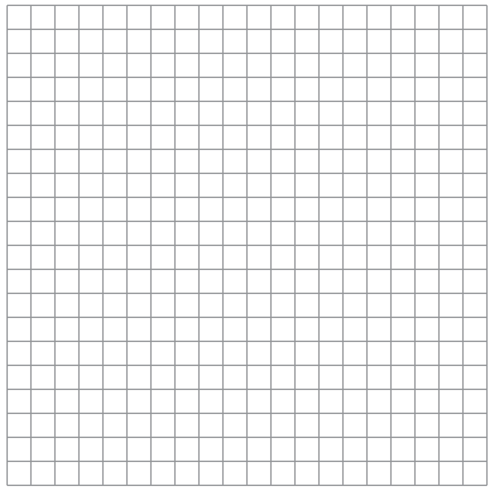
6. Which quadrilateral would Tomas choose and why? Determine the area of the entire figure.

PROBLEM 3 Party Time!

1. Use the coordinate plane shown to draw a floor plan for a space that would allow Marissa to invite 100 people including herself to her graduation party, given the rule of thumb for maximum occupancy. Two coordinates of the location are provided measured in feet: (25, 30) and (65, 75). Show your work.



Be prepared to share your solutions and methods.

Looking at Something Familiar in a New Way

3.2

Area and Perimeter of Triangles on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of triangles on the coordinate plane.
- Determine the area of triangles on the coordinate plane.
- Explore the effects that doubling the area has on the properties of a triangle.

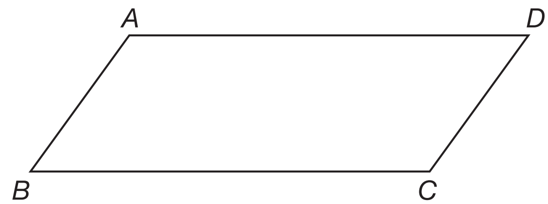
One of the most famous stretches of ocean in the Atlantic is an area that stretches between the United States, Puerto Rico, and Bermuda known as the Bermuda Triangle.

A heavily traveled area by planes and ships, it has become famous because of the many stories about ships and planes lost or destroyed as they moved through the Triangle.

For years, the Bermuda Triangle was suspected of having mysterious, supernatural powers that fatally affected all who traveled through it. Others believe natural phenomena, such as human error and dangerous weather, are to blame for the incidents.

PROBLEM 1 Determining the Area of a Triangle

1. The formula for calculating the area of a triangle can be determined from the formula for the area of a parallelogram.



   a. Explain how the formula for the area of a triangle is derived using the given parallelogram.

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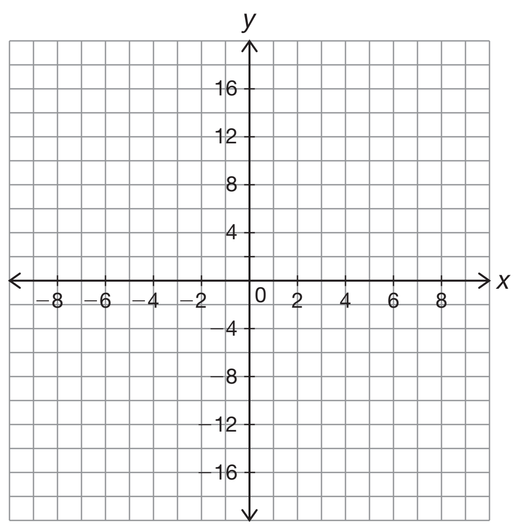
   b. Write the formula for the area of a triangle.

2. Graph triangle *ABC* with vertices *A*(−7.5, 2), *B*(−5.5, 13), and *C*(2.5, 2). Then, determine its perimeter.



3. Determine the area of triangle *ABC*.

   a. What information is needed about triangle *ABC* to determine its area?

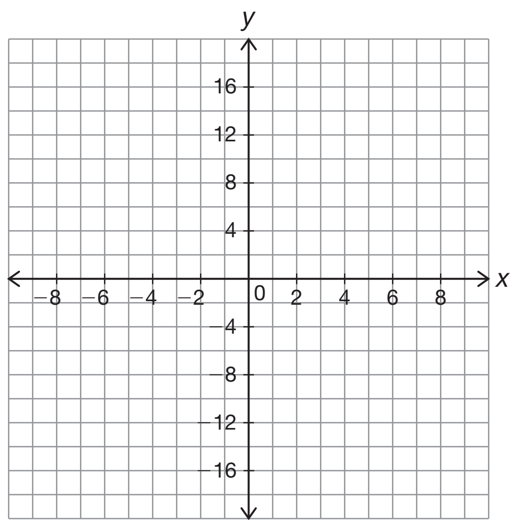
   b. Arlo says that line segment *AB* can be used as the height. Trisha disagrees and says that line segment *BC* can be used as the height. Randy disagrees with both of them and says that none of the line segments that make up the triangle can be used as the height. Who is correct? Explain your reasoning.

   c. Draw a line segment that represents the height of triangle *ABC*. Label the line segment *BD*. Then, determine the height of triangle *ABC*.

   d. Determine the area of triangle *ABC*.

4. Perform one or more transformations to help you determine the perimeter and the area more efficiently.

   a. Transform triangle *ABC* on the coordinate plane. Label the image *A′B′C′*. Describe the transformation(s) completed and explain your reasoning.



   b. Determine the perimeter of triangle *A′B′C′*.

c. Determine the area of triangle $A'B'C'$. Be sure to label the height on the coordinate plane as line segment $B'D'$.

5. Compare the perimeters and the areas of triangles ABC and $A'B'C'$.

a. What do you notice about the perimeters and the areas of both triangles?

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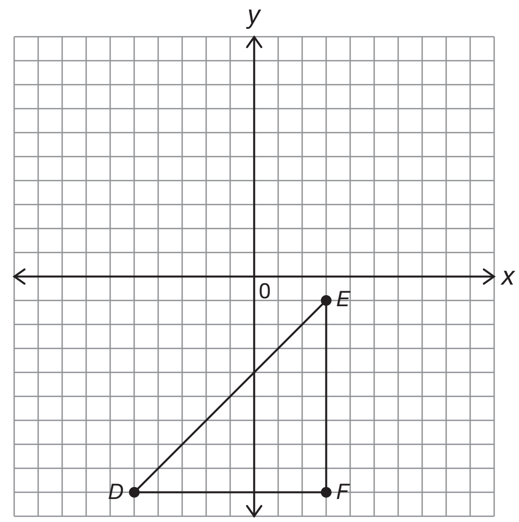
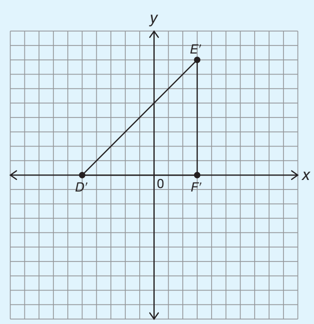
b. Use what you know about transformations to explain why this occurs.

6. Mr. Young gives his class triangle *DEF* and asks them to determine the area and perimeter.

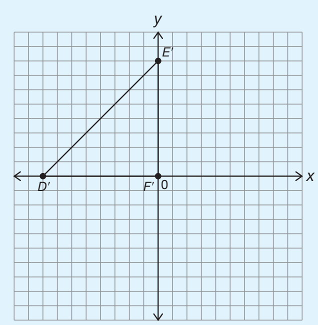


Four of his students decide to first transform the figure and then determine the perimeter and the area. Their transformations are shown.

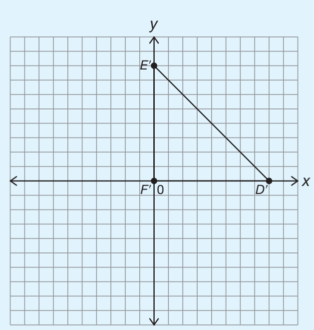
Michael



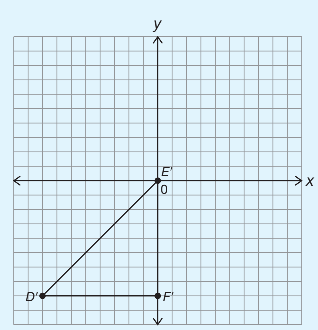
Angelica



Juan



Isabel



a. Describe the transformation(s) each student made to triangle *DEF*.

b. Whose method is most efficient? Explain your reasoning.

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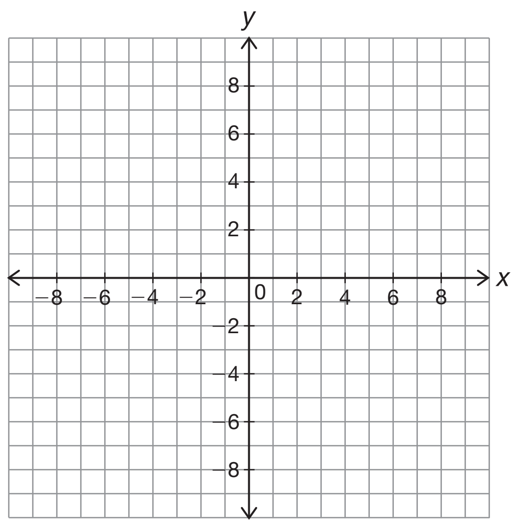
c. What do you know about the perimeters and areas of all the students' triangles? Explain your reasoning.

PROBLEM 2 Which Way Is Up?

1. Graph triangle *ABC* with vertices *A*(2, 5), *B*(10, 9), and *C*(6, 1). Determine the perimeter.



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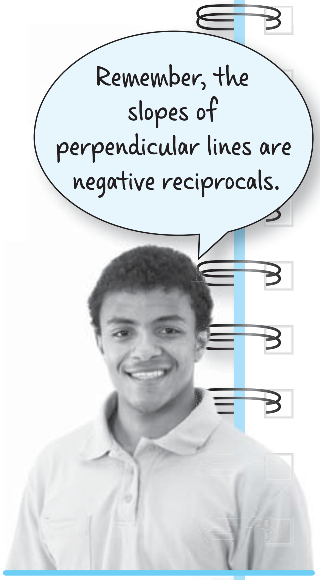
2. To determine the area, you will need to determine the height. How will determining the height of this triangle be different from determining the height of the triangle in Problem 1?

To determine the height of this triangle, you must first determine the endpoints of the height. Remember that the height must always be perpendicular to the base.



Let's use $\overline{AC}$ as the base of triangle *ABC*. Determine the coordinates of the endpoints of height $\overline{BD}$.

Calculate the slope of the base.	$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 5}{6 - 2} = \frac{-4}{4} = -1$
Determine the slope of the height.	$m = 1$
Determine the equation of the base.	Base $\overline{AC}$ has a slope of -1 and passed through point *A* (5, 2). $(y - y_1) = m(x - x_1)$ $(y - 2) = -1(x - 5)$ $y = -x + 7$
Determine the equation of the height.	Height $\overline{BD}$ has a slope of 1 and passed through point *B* (10, 9). $(y - y_1) = m(x - x_1)$ $(y - 9) = 1(x - 10)$ $y = x - 1$
Solve the system of equations to determine the coordinates of the point of intersection.	$x - 1 = -x + 7$ $y = x - 1$ $2x = 8$ $y = 4 - 1$ $x = 4$ $y = 3$ The coordinates of point *D* are (4, 3).

3. Graph the point of intersection on the coordinate plane and label it point *D*. Draw line segment *BD* to represent the height.

4. Determine the area of triangle *ABC*.

   a. Determine the length of height $\overline{BD}$.

   b. Determine the area of triangle *ABC*.

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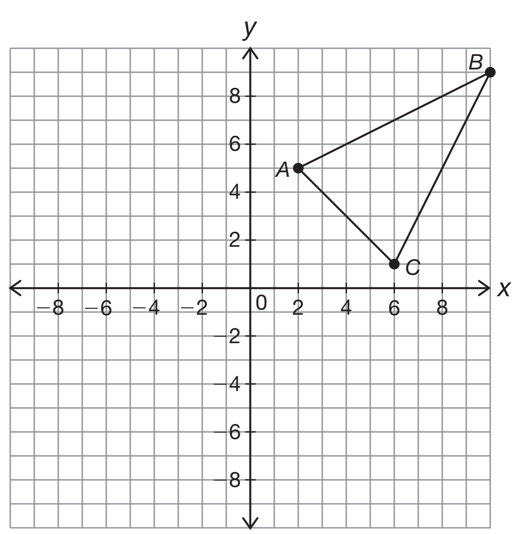
5. You know that any side of a triangle can be the base of the triangle. Predict whether using a different side as the base will result in a different area of the triangle. Explain your reasoning.

Let's consider your prediction.

6. Triangle *ABC* is given on the coordinate plane. This time, let's consider side $\overline{AB}$ as the base.



a. Let point *D* represent the intersection point of the height, $\overline{CD}$, and the base. Determine the coordinates of point *D*.

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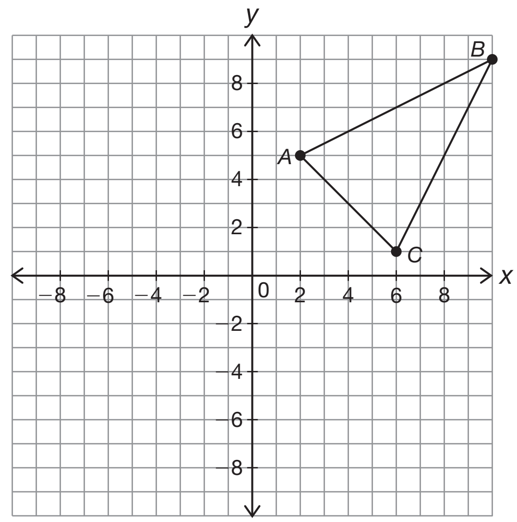
b. Determine the height of triangle *ABC*.

c. Determine the area of triangle *ABC*.

7. Triangle *ABC* is graphed on the coordinate plane. Determine the area of triangle *ABC* using $\overline{BC}$ as the base.



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8. Compare the three areas you determined for triangle *ABC*. Was your prediction in Question 5 correct?

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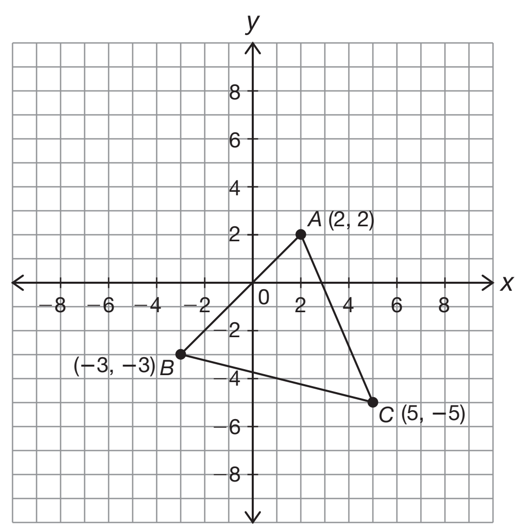
PROBLEM 3 It's a Dog's Life

Joseph plans to fence in a corner of his property so his dog can exercise there. Consider the triangular space shown. Each of the three corners of the space is labeled with coordinates and helps define the dimensions, in feet, of the fenced portion of land.



1. Fencing costs $15 per linear foot. How much will this project cost Joseph? Show your work.

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2. Calculate the amount of space Joseph's dog will have to exercise. Show your work.

3

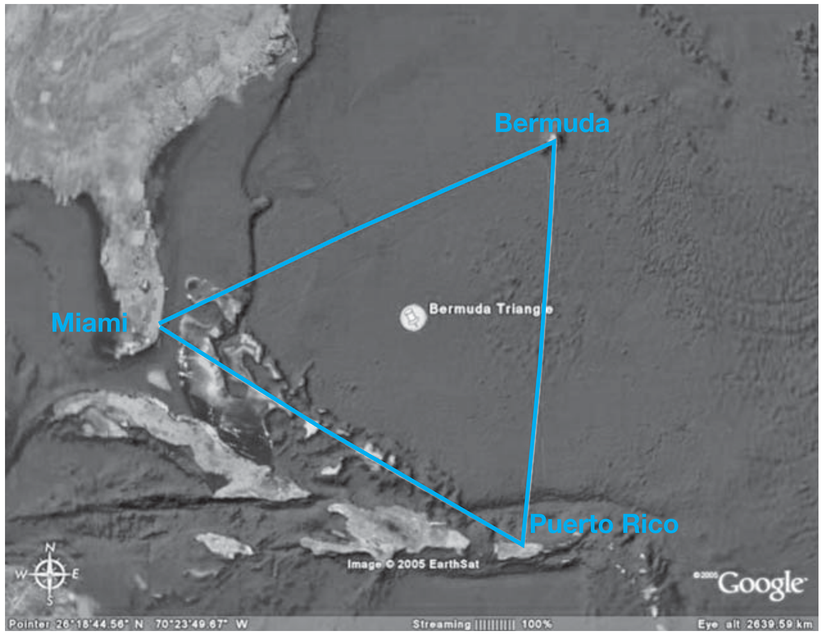
3. Compare your answer to Question 2 with your classmates' answers. Was your solution path the same or different from your classmates' solution paths?

3

4. Describe how transformations could be used to make the calculations more efficient.

5. If the same vertical and horizontal translations were performed on the three vertices of the triangle, describe how this would affect the perimeter and the area of the triangle.

PROBLEM 4 The Bermuda Triangle

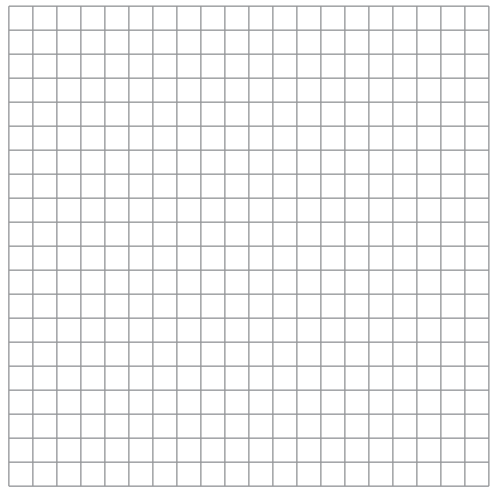


The Bermuda Triangle is an imaginary triangle connecting Miami, Florida, to San Juan, Puerto Rico, to Bermuda. It has a rich history of suspected paranormal activities, which include the disappearances of boats and aircraft.

Consider these approximate measurements:

- The distance from Miami to San Juan is 1060 miles.
- The distance from Miami to Bermuda is 1037 miles.
- The perimeter of the Bermuda Triangle is 3078 miles.
- The Bermuda Triangle is a region of 454,000 square miles.

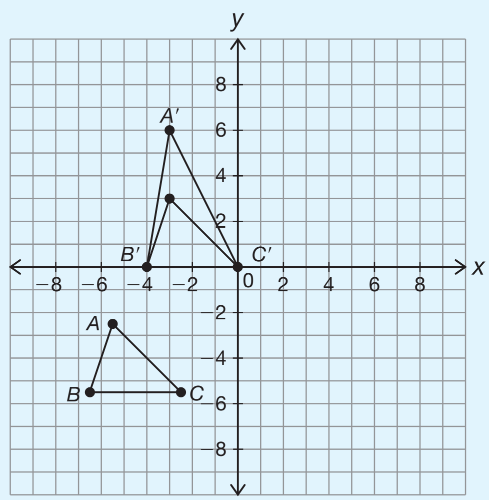
Place the Bermuda Triangle on the coordinate plane and provide the coordinates for each of the three vertices. Show your work.



Talk the Talk

1. Emilio's class is given triangle *ABC*. Their teacher asks them to double the area of this triangle by manipulating the height. They must identify the coordinates of the new point, *A*′, and then determine the area. Emilio decides to first translate the triangle so it sits on grid lines to make his calculations more efficient. His work is shown.

Emilo



The coordinates of point A' are $(-3, 6)$.

The area of triangle $A'B'C'$ is 12 square units.

$$A = \frac{1}{2}(4)(6)$$

$$A = 12$$

Emilio is shocked to learn that he got this answer wrong. Explain to Emilio what he did wrong. Determine the correct answer for this question.

Be prepared to share your solutions and methods.

3

Grasshoppers Everywhere!

3.3

Area and Perimeter of Parallelograms on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter of parallelograms on a coordinate plane.
- Determine the area of parallelograms on a coordinate plane.
- Explore the effects that doubling the area has on the properties of a parallelogram.

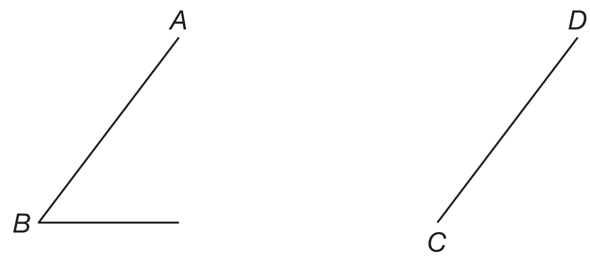
You wouldn't think that grasshoppers could be dangerous. But they can damage farmers' crops and destroy vegetation. In 2003, a huge number of grasshoppers invaded the country of Sudan, affecting nearly 1700 people with breathing problems.

Grasshopper invasions have been recorded in North America, Europe, the Middle East, Africa, Asia, and Australia. One of the largest swarms of grasshoppers—known as a "cloud" of grasshoppers—covered almost 200,000 square miles!

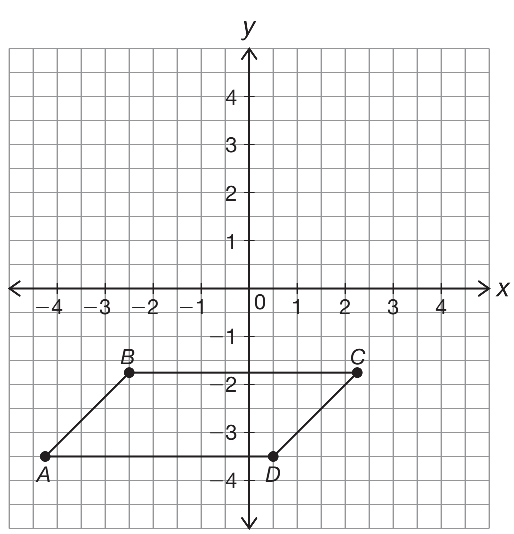
PROBLEM 1 Rectangle or . . . ?

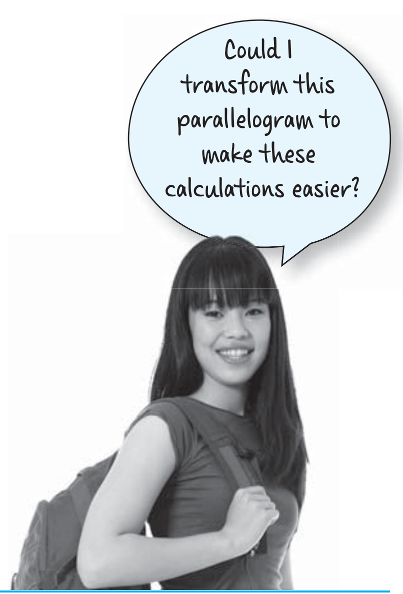
You know the formula for the area of a parallelogram. The formula, $A = bh$, where A represents the area, b represents the length of the base, and h represents the height, is the same formula that is used when determining the area of a rectangle. But how can that be if they are different shapes?

1. Use the given parallelogram to explain how the formula for the area of a parallelogram and the area of a rectangle can be the same.



2. Analyze parallelogram *ABCD* on the coordinate plane.





a. Determine the perimeter of parallelogram *ABCD*.

b. To determine the area of parallelogram *ABCD*, you must first determine the height. Describe how to determine the height of parallelogram *ABCD*.

c. Ms. Finch asks her class to identify the height of parallelogram *ABCD*. Peter draws a perpendicular line from point *B* to $\overline{AD}$, saying that the height is represented by $\overline{BE}$. Tonya disagrees. She draws a perpendicular line from point *D* to $\overline{BC}$, saying that the height is represented by $\overline{DF}$. Who is correct? Explain your reasoning.

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d. Determine the height of parallelogram *ABCD*.

e. Determine the area of parallelogram *ABCD*.

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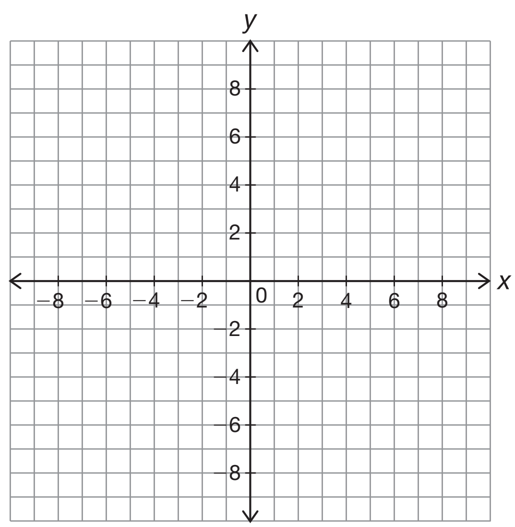
PROBLEM 2 Stand Up Straight!

1. Graph parallelogram *ABCD* with vertices *A* (1, 1), *B* (7, −7), *C* (8, 0), and *D* (2, 8). Determine the perimeter.



2. Determine the area of parallelogram *ABCD*.

a. Using $\overline{CD}$ as the base, how will determining the height of this parallelogram be different from determining the height of the parallelogram in Problem 1?

b. Using $\overline{CD}$ as the base, explain how you will locate the coordinates of point *E*, the point where the base and height intersect.

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c. Determine the coordinates of point *E*. Label point *E* on the coordinate plane.

d. Determine the height of parallelogram *ABCD*.

e. Determine the area of parallelogram *ABCD*.

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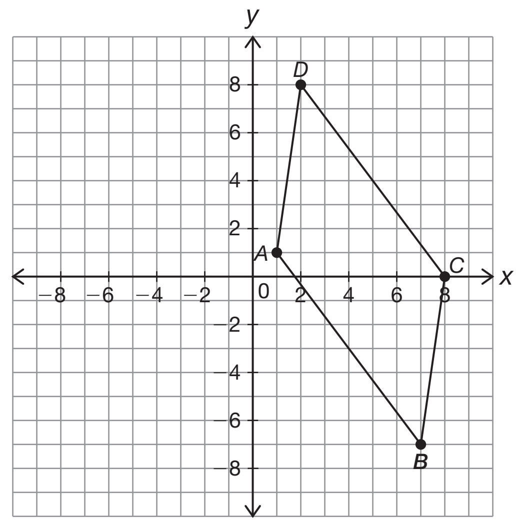
3. You determined earlier that any side of a parallelogram can be thought of as the base. Predict whether using a different side as the base will result in a different area of the parallelogram. Explain your reasoning.

Let's consider your prediction.

4. Parallelogram *ABCD* is given on the coordinate plane. This time, let's consider side $\overline{BC}$ as the base.



a. Let point *E* represent the intersection point of the height, $\overline{AE}$, and the base. Determine the coordinates of point *E*.

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b. Determine the area of parallelogram *ABCD*.

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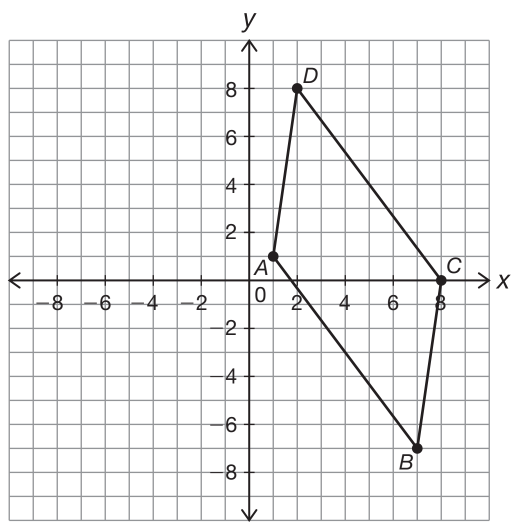
5. Compare the area you calculated in Question 4, part (b) with the area you calculated in Question 2, part (e). Was your prediction in Question 3 correct? Explain why or why not.

PROBLEM 3 Time for a Little Boxing

In the previous problem, you learned one method to calculate the area of a parallelogram.

1. Summarize the steps to calculate the area of a parallelogram using the method presented in Problem 2.

Now, let's explore another method that you can use to calculate the area of a parallelogram. Consider parallelogram *ABCD* from Problem 2 Question 1.



2. Sketch a rectangle that passes through points *A*, *B*, *C*, *D* so that each side of the rectangle passes through one point of the parallelogram and all sides of the rectangle are either horizontal or vertical. Label the vertices of your rectangle as *W*, *X*, *Y*, and *Z*.

3. Determine the coordinates of *W*, *X*, *Y*, and *Z*. Explain how you calculated each coordinate.

4. Calculate the area of each figure. Show all your work.

   a. rectangle *WXYZ*

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   b. triangle *ABZ*

   c. triangle *BCY*

   d. triangle *CDX*

   e. triangle *DAW*

   f. parallelogram *ABCD*

5. Compare the area you calculated in Question 4, part (f) to the area that you calculated in Problem 2. What do you notice?

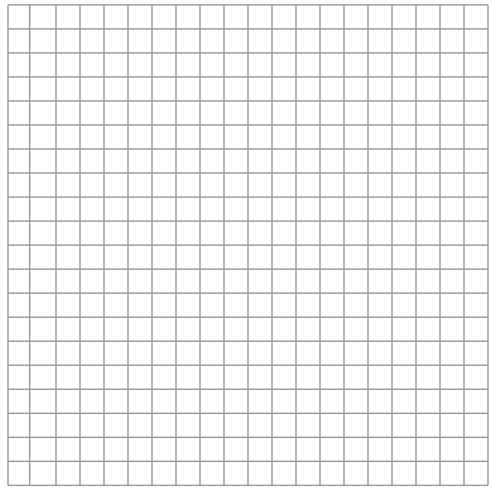
6. Summarize the steps to calculate the area of a parallelogram using the method presented in Problem 3.

3

7. Which method for calculating the area of a parallelogram do you prefer? Why?

8. Do you think the rectangle method will work to calculate the area of a triangle? Explain your reasoning.

9. On the coordinate plane, draw your own triangle. Use the rectangle method to calculate the area of the triangle that you drew.



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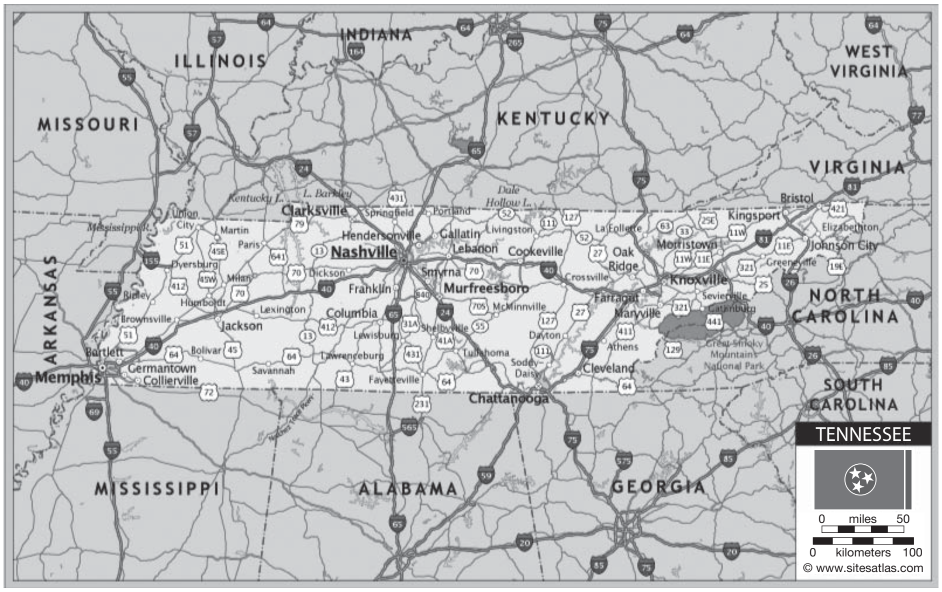
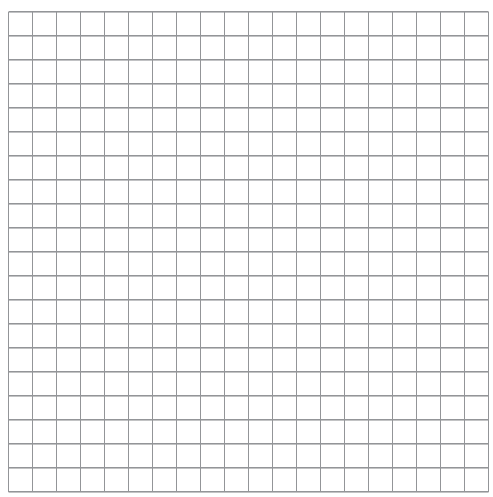
PROBLEM 4 Tennessee



1. Transfer the state of Tennessee into the coordinate plane shown.



2. Suppose Tennessee had an outbreak of killer grasshoppers. One scientist says that it is necessary to spray insecticide over the entire state. Determine the approximate area that needs to be treated. Explain how you found your answer.

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3. Suppose one tank of insect spray costs $600, and the tank covers 1000 square miles. How much will this project cost? Show your work.

4. A second scientist says it would only be necessary to spray the perimeter of the state. The type of spray needed to do this job is more concentrated and costs $2000 per tank. One tank of insect spray covers 100 linear miles. How much will this project cost? Show your work.

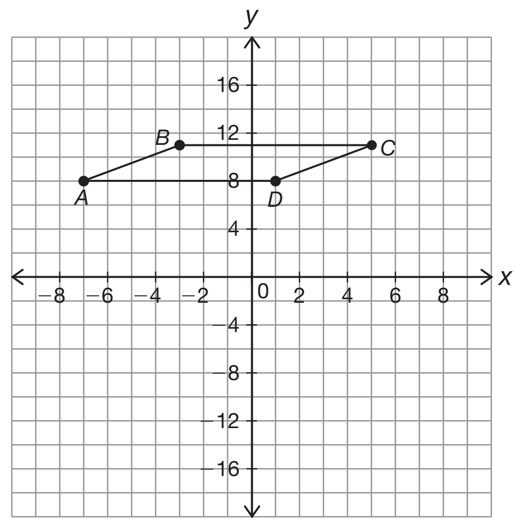
3

5. Which method of spraying is more cost efficient?

6. If the population of Tennessee is approximately 153.9 people per square mile, how many people live in the state?

Talk the Talk

1. Parallelogram *ABCD* is given. Double the area of parallelogram *ABCD* by manipulating the height. Label the image, identify the coordinates of the new point(s), and determine the area.



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Be prepared to share your solutions and methods.

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3.4

Leavin' on a Jet Plane

Area and Perimeter of Trapezoids on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeter and the area of trapezoids and hexagons on a coordinate plane.
- Determine the perimeter of composite figures on the coordinate plane.

KEY TERMS

- bases of a trapezoid
- legs of a trapezoid

How can you make a building withstand an earthquake? The ancient Incas figured out a way—by making trapezoidal doors and windows.

The Inca Empire expanded along the South American coast—an area that experiences a lot of earthquakes—from the 12th through the late 15th century.

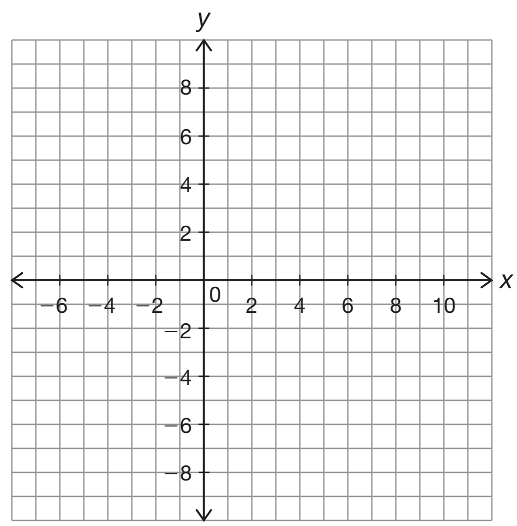
One of the most famous of Inca ruins is Machu Picchu in Peru. There you can see the trapezoidal doors and windows—tilting inward from top to bottom to better withstand the seismic activity.

PROBLEM 1 Well, It's the Same, But It's Also Different!

So far, you have determined the perimeter and the area of parallelograms—including rectangles and squares. Now, you will move on to trapezoids.

1. Plot each point in the coordinate plane shown:
   - $A(-5, 4)$
   - $B(-5, -4)$
   - $C(6, -4)$
   - $D(0, 4)$

   Then, connect the points in alphabetical order.



2. Explain how you know that the quadrilateral you graphed is a trapezoid.

The trapezoid is unique in the quadrilateral family because it is a quadrilateral that has *exactly* one pair of parallel sides. The parallel sides are known as the **bases of the trapezoid**, while the non-parallel sides are called the **legs of the trapezoid**.

3. Using the trapezoid you graphed, identify:

   a. the bases.

   b. the legs.

4. Analyze trapezoid *ABCD* that you graphed on the coordinate plane.

   a. Describe how you can determine the perimeter of trapezoid *ABCD* without using the Distance Formula.

   b. Determine the perimeter of trapezoid *ABCD* using the strategy you described in part (a). First, perform a transformation of trapezoid *ABCD* on the coordinate plane and then calculate the perimeter of the image.

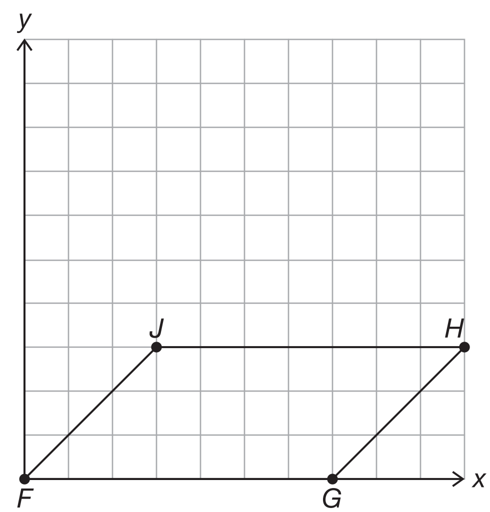
3

PROBLEM 2 Using What You Know

So, what similarities are there between determining the area of a parallelogram and determining the area of a trapezoid?

Recall that the formula for the area of a parallelogram is $A = bh$, where b represents the base and h represents the height. As you know, a parallelogram has both pairs of opposite sides parallel. But what happens if you divide the parallelogram into two congruent trapezoids?

1. Analyze parallelogram *FGHJ* on the coordinate plane.



a. Divide parallelogram *FGHJ* into two congruent trapezoids.

b. Label the two vertices that make up the two congruent trapezoids.

c. Label the bases that are congruent to each other. Label one pair of bases b_1 and the other pair b_2.

d. Now write a formula for the area of the entire geometric figure. Make sure you use the bases you labeled and do not forget the height.

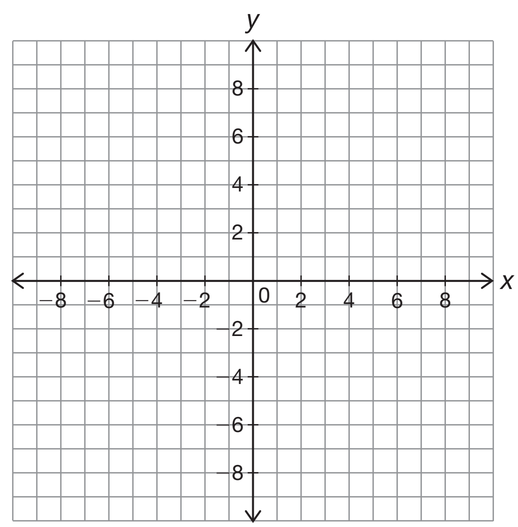
e. Now write the formula for the area for *half* of the entire figure.

2. What can you conclude about the area formula of a parallelogram and the area formula of a trapezoid? Why do you think this connection exists?

3. Plot each point on the coordinate plane shown:

- $Q(-2, -2)$
- $R(5, -2)$
- $S(5, 2)$
- $T(1, 2)$

Then, connect the points in alphabetical order.



4. Determine the area of trapezoid *QRST*. Describe the strategy or strategies you used to determine your answer.

3

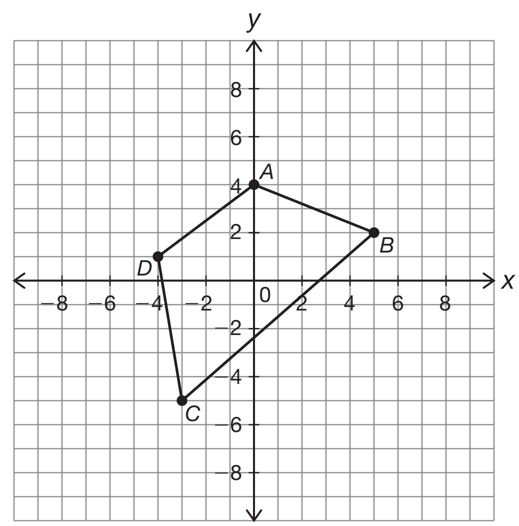
PROBLEM 3 Box Up That Trapezoid

In the previous lesson, you learned a method to calculate the area of a parallelogram using a rectangle. Can the same method be used to calculate the area of a trapezoid?

1. Summarize the steps to calculate the area of a parallelogram using the rectangle method.

3

Consider trapezoid *ABCD*.



2. Sketch a rectangle that passes through points *A*, *B*, *C*, *D* so that each side of the rectangle passes through one point of the trapezoid and all sides of the rectangle are either horizontal or vertical. Label the vertices of your rectangle as *W*, *X*, *Y*, and *Z*.

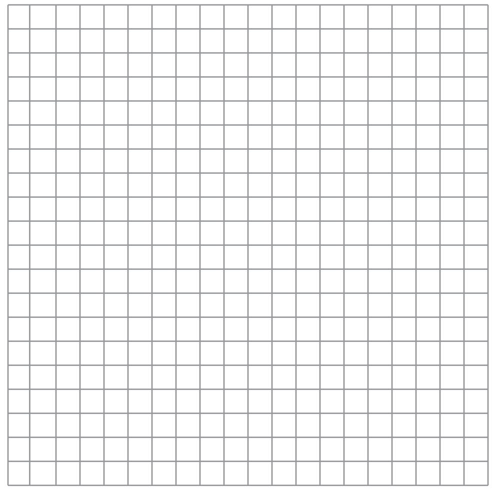
3. Determine the coordinates of *W*, *X*, *Y*, and *Z*. Explain how you calculated each coordinate.

4. Calculate the area of trapezoid *ABCD* using the rectangle method. Show all your work.

3

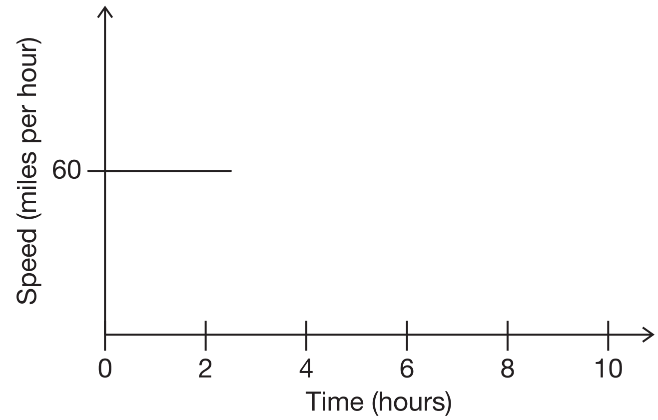
5. Do you think the rectangle method will work to calculate the area of any trapezoid? Explain your reasoning.

6. On the coordinate plane, draw your own trapezoid. Use the rectangle method to calculate the area of the trapezoid that you drew.



PROBLEM 4 Jets and Trapezoids!

The graph shows the constant speed of a car on the highway over the course of 2.5 hours.



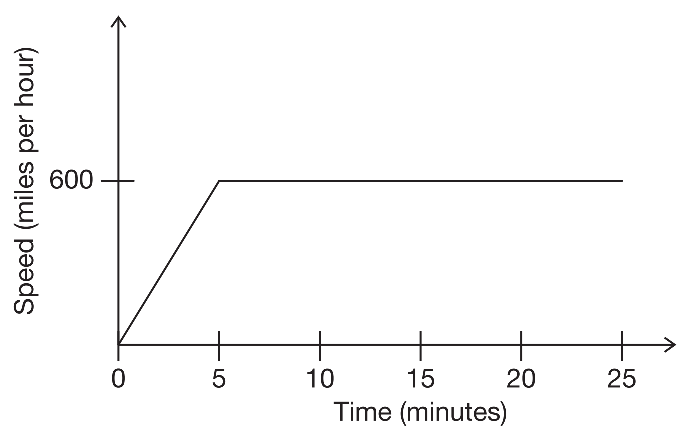
3

1. Describe how you could calculate the distance the car traveled in 2.5 hours using what you know about area.

2. How far did the car travel in 2.5 hours?

The graph you used is called a velocity-time graph. In a velocity-time graph, the area under the line or curve gives the distance.

The graph shown describes the speed and the time of a passenger jet's ascent.



3. How can you use the graph to determine the distance the jet has traveled in 25 minutes?

4. What shape is the region on the graph enclosed by the line segments?

3

5. Determine the distance the jet has traveled in 25 minutes. Show your work.

6. Determine the distance the jet has traveled in 5 minutes. Show your work.

Be prepared to share your solutions and methods.

Composite Figures on the Coordinate Plane

3.5

Area and Perimeter of Composite Figures on the Coordinate Plane

LEARNING GOALS

In this lesson, you will:

- Determine the perimeters and the areas of composite figures on a coordinate plane.
- Connect transformations of geometric figures with number sense and operations.
- Determine the perimeters and the areas of composite figures using transformations.

KEY TERM

- composite figures

Did you ever think about street names? How does a city or town decide what to name their streets?

Some street names seem to be very popular. In the United States, almost every town has a Main Street. But in France, there is literally a Victor Hugo Street in every town!

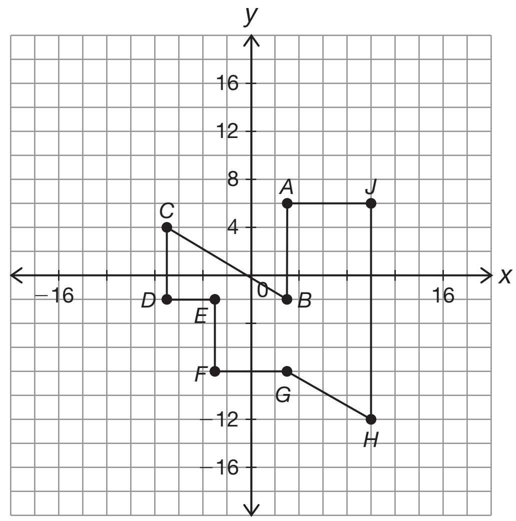
Victor Hugo was a French writer. He is best known for writing the novels *Les Miserables* and *Notre-Dame de Paris*, better known as *The Hunchback of Notre Dame* in English.

If you were in charge of naming the streets in your town, what names would you choose? Would you honor any people with their own streets?

PROBLEM 1 Breakin' It Down

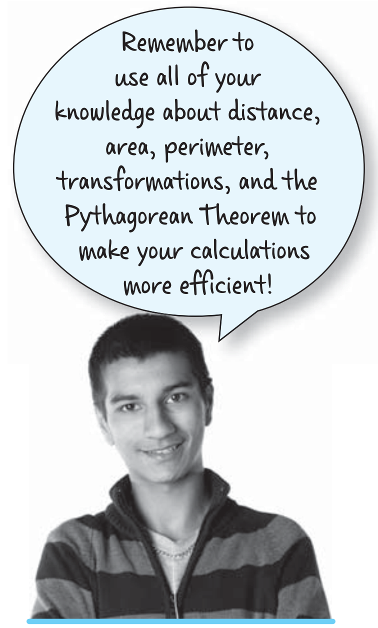
Now that you have determined the perimeters and the areas of various quadrilaterals and triangles, you can use this knowledge to determine the perimeters and the areas of *composite figures*. A **composite figure** is a figure that is formed by combining different shapes.

1. A composite figure is graphed on the coordinate plane shown.

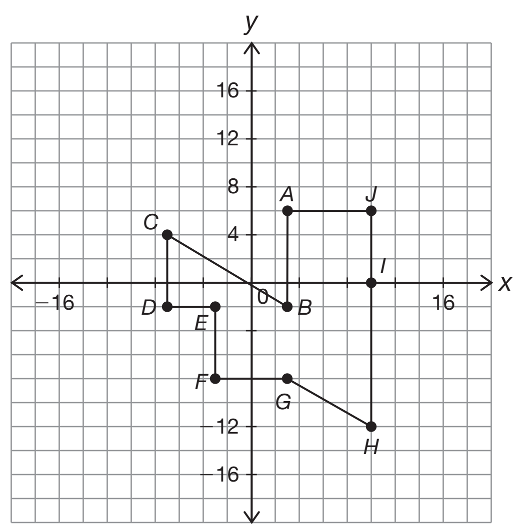


Determine the perimeter of the composite figure. Round to the nearest tenth if necessary.

2. Draw line segments on the composite figure to divide the figure. Determine the area of the composite figure. Round to the nearest tenth if necessary.



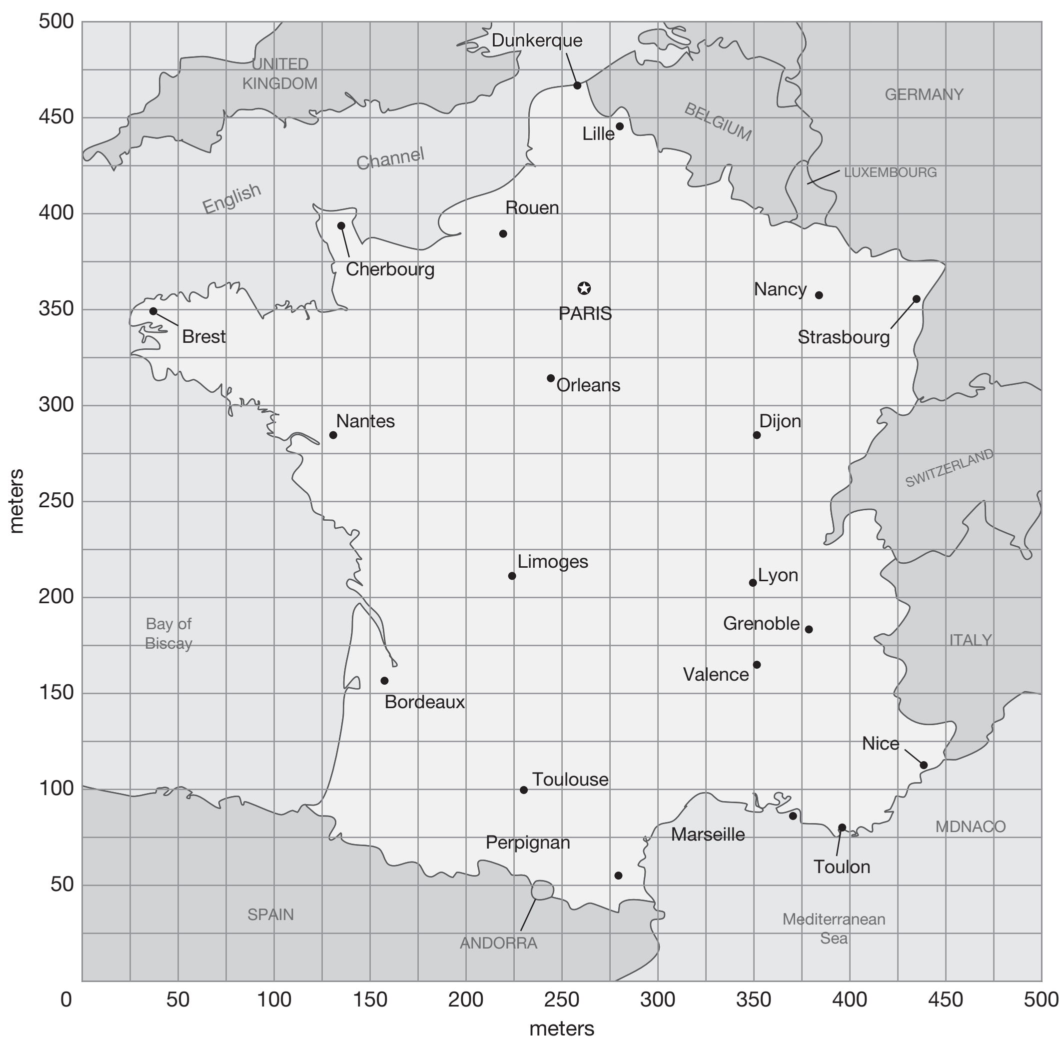
3. Draw line segments on the composite figure to divide the figure differently from how you divided it in Question 2. Determine the area of the composite figure. Round to the nearest tenth if necessary.



4. How does the area in Question 2 compare to the area in Question 3? Explain your reasoning.

PROBLEM 2 Is France Hexagonal?

1. Draw a hexagon to approximate the shape of France. Use the hexagon for Questions 2 and 3.



2. Which of the following statements is true?

- The coastline of France is greater than 5000 kilometers.
- The coastline of France is less than 5000 kilometers.
- The coastline of France is approximately 5000 kilometers.

3

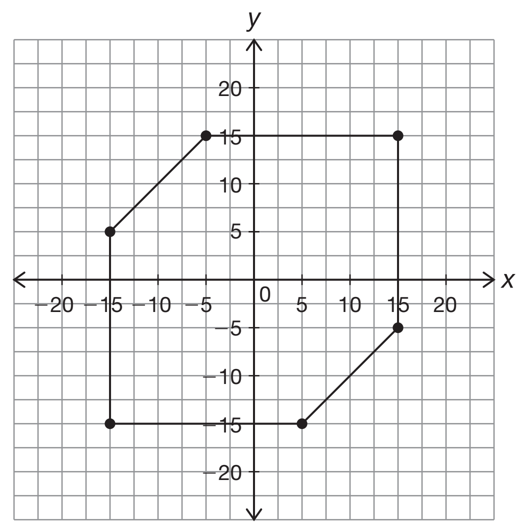
3. Which of the following statements is true?

- The area of France is greater than 1,000,000 square kilometers.
- The area of France is less than 1,000,000 square kilometers.
- The area of France is approximately 1,000,000 square kilometers.

4. If the population of France is approximately 118.4 people per square mile, how many people live in the country of France?

Talk the Talk

Draw line segments on the composite figure to divide the figure into familiar shapes two different ways, and then determine the area of the composite figure each way to show the area remains unchanged.



Be prepared to share your solutions and methods.

Chapter 3 Summary

KEY TERMS

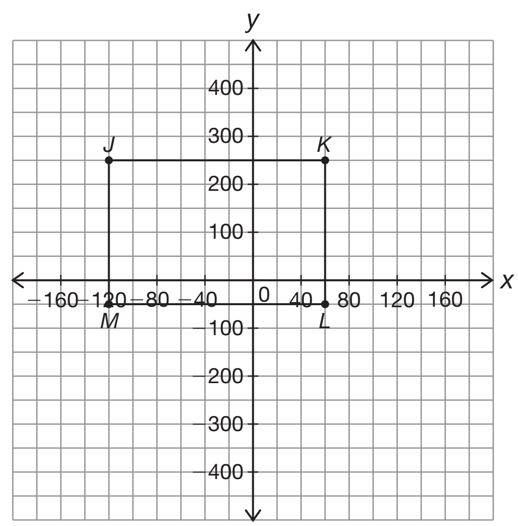
- bases of a trapezoid (3.4)
- legs of a trapezoid (3.4)
- composite figure (3.5)

3.1 Determining the Perimeter and Area of Rectangles and Squares on the Coordinate Plane

The perimeter or area of a rectangle can be calculated using the distance formula or by counting the units of the figure on the coordinate plane. When using the counting method, the units of the *x*-axis and *y*-axis must be considered to count accurately.

Example

Determine the perimeter and area of rectangle *JKLM*.



The coordinates for the vertices of rectangle *JKLM* are *J*(−120, 250), *K*(60, 250), *L*(60, −50), and *M*(−120, −50).

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

$JK = 60 - (-120)$
$= 180$

$KL = 250 - (-50)$
$= 300$

$P = JK + KL + LM + JM$
$= 180 + 300 + 180 + 300$
$= 960$

The perimeter of rectangle *JKLM* is 960 units.

$A = bh$
$= 180(300)$
$= 54{,}000$

The area of rectangle *JKLM* is 54,000 square units.

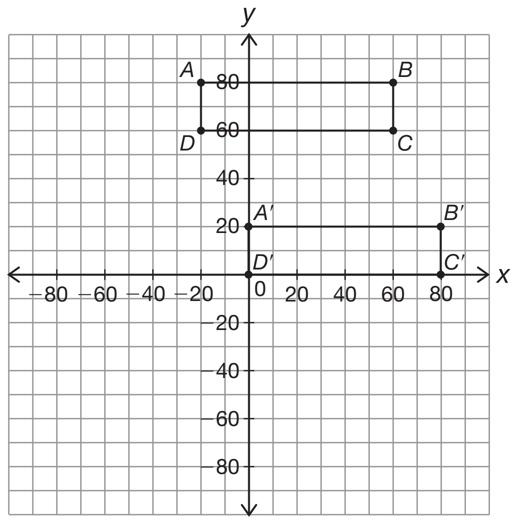
3.1

Using Transformations to Determine the Perimeter and Area of Geometric Figures

If a rigid motion is performed on a geometric figure, not only are the pre-image and the image congruent, but both the perimeter and area of the pre-image and the image are equal. Knowing this makes solving problems with geometric figures more efficient.

Example

Determine the perimeter and area of rectangle *ABCD*.



The vertices of rectangle *ABCD* are $A(-20, 80)$, $B(60, 80)$, $C(60, 60)$, and $D(-20, 60)$. To translate point *D* to the origin, translate *ABCD* to the right 20 units and down 60 units. The vertices of rectangle $A'B'C'D'$ are $A'(0, 20)$, $B'(80, 20)$, $C'(80, 0)$, and $D'(0, 0)$.

Because the sides of the rectangle lie on grid lines, subtraction can be used to determine the lengths.

$$A'D' = 20 - 0 = 20 \qquad C'D' = 80 - 0 = 80$$

$$\begin{aligned} P &= A'B' + B'C' + C'D' + A'D' \\ &= 80 + 20 + 80 + 20 \\ &= 200 \end{aligned}$$

The perimeter of rectangle $A'B'C'D'$ and, therefore, the perimeter of rectangle *ABCD*, is 200 units.

$$\begin{aligned} A &= bh \\ &= 20(80) \\ &= 1600 \end{aligned}$$

The area of rectangle $A'B'C'D'$ and, therefore, the area of rectangle *ABCD*, is 1600 square units.

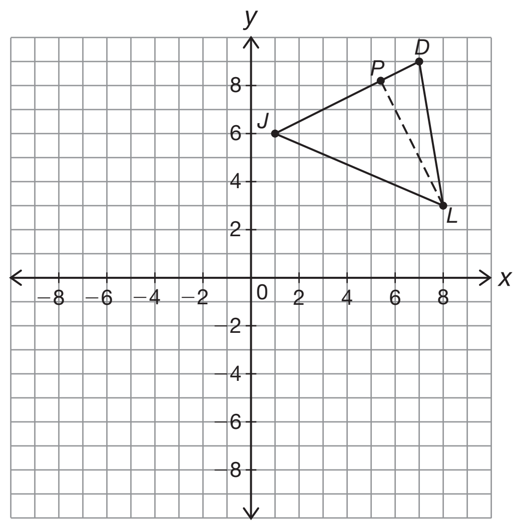
3

3.2 Determining the Perimeter and Area of Triangles on the Coordinate Plane

The formula for the area of a triangle is half the area of a rectangle. Therefore, the area of a triangle can be found by taking half of the product of the base and the height. The height of a triangle must always be perpendicular to the base. On the coordinate plane, the slope of the height is the negative reciprocal of the slope of the base.

Example

Determine the perimeter and area of triangle *JDL*.



The vertices of triangle *JDL* are *J*(1, 6), *D*(7, 9), and *L*(8, 3).

$$JD = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(7 - 1)^2 + (9 - 6)^2} = \sqrt{6^2 + 3^2} = \sqrt{36 + 9} = \sqrt{45} = 3\sqrt{5}$$

$$DL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(8 - 7)^2 + (3 - 9)^2} = \sqrt{1^2 + (-6)^2} = \sqrt{1 + 36} = \sqrt{37}$$

$$LJ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(1 - 8)^2 + (6 - 3)^2} = \sqrt{(-7)^2 + 3^2} = \sqrt{49 + 9} = \sqrt{58}$$

$$P = JD + DL + LJ = 3\sqrt{5} + \sqrt{37} + \sqrt{58} \approx 20.4$$

The perimeter of triangle *JDL* is approximately 20.4 units.

3

To determine the area of the triangle, first determine the height of triangle *JDL*.

Slope of $\overline{JD}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{9 - 6}{7 - 1}$

$= \frac{3}{6}$

$= \frac{1}{2}$

Slope of $\overline{PL}$: $m = -2$

Equation of $\overline{JD}$: $(y - y_1) = m(x - x_1)$

$y - 6 = \frac{1}{2}(x - 1)$

$y = \frac{1}{2}x + 5\frac{1}{2}$

Equation of $\overline{PL}$: $(y - y_1) = m(x - x_1)$

$y - 3 = -2(x - 8)$

$y = -2x + 19$

Intersection of $\overline{JD}$ and $\overline{PL}$, or *P*: $\frac{1}{2}x + 5\frac{1}{2} = -2x + 19$

$\frac{1}{2}x + 2x = 19 - 5\frac{1}{2}$

$2\frac{1}{2}x = 13\frac{1}{2}$

$x = 5.4$

$y = -2(5.4) + 19$

$y = 8.2$

The coordinates of *P* are (5.4, 8.2).

Height of triangle *JDL*: $PL = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(8 - 5.4)^2 + (3 - 8.2)^2}$

$= \sqrt{(2.6)^2 + (-5.2)^2}$

$= \sqrt{33.8}$

≈ 5.8

Area of triangle *JDL*: $A = \frac{1}{2}bh$

$= \frac{1}{2}(JD)(PL)$

$= \frac{1}{2}(3\sqrt{5})(\sqrt{33.8})$

$= \frac{1}{2}(3\sqrt{169})$

$= 19.5$

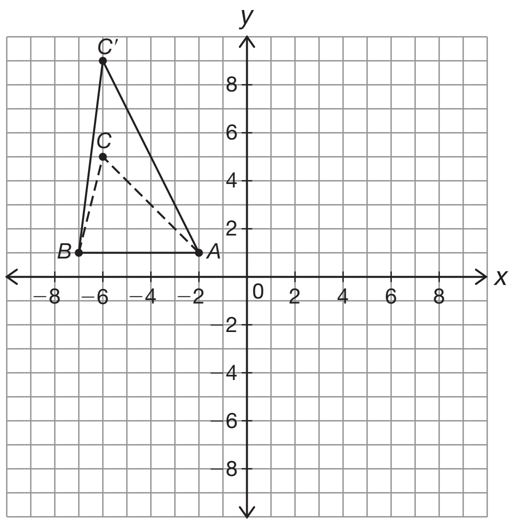
The area of triangle *JDL* is 19.5 square units.

3.2 Doubling the Area of a Triangle

To double the area of a triangle, only the length of the base or the height of the triangle need to be doubled. If both the length of the base and the height are doubled, the area will quadruple.

Example

Double the area of triangle *ABC* by manipulating the height.



Area of *ABC*

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(4)$

$= 10$

Area of *ABC′*

$A = \frac{1}{2}bh$

$= \frac{1}{2}(5)(8)$

$= 20$

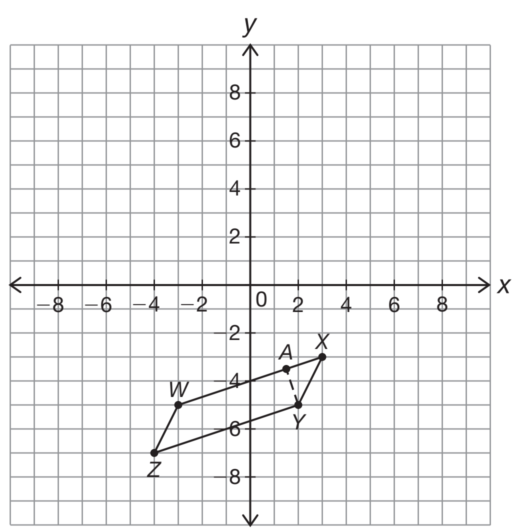
By doubling the height, the area of triangle *ABC′* is double the area of triangle *ABC*.

3.3 Determining the Perimeter and Area of Parallelograms on the Coordinate Plane

The formula for calculating the area of a parallelogram is the same as the formula for calculating the area of a rectangle: $A = bh$. The height of a parallelogram is the length of a perpendicular line segment from the base to a vertex opposite the base.

Example

Determine the perimeter and area of parallelogram *WXYZ*.



The vertices of parallelogram $WXYZ$ are $W(-3, -5)$, $X(3, -3)$, $Y(2, -5)$, and $Z(-4, -7)$.

$$WX = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(3 - (-3))^2 + (-3 - (-5))^2}$$
$$= \sqrt{6^2 + 2^2}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10}$$

$$YZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4 - 2)^2 + (-7 - (-5))^2}$$
$$= \sqrt{(-6)^2 + (-2)^2}$$
$$= \sqrt{40}$$
$$= 2\sqrt{10}$$

$$WZ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(-4 - (-3))^2 + (-7 - (-5))^2}$$
$$= \sqrt{(-1)^2 + (-2)^2}$$
$$= \sqrt{5}$$

$$XY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - 3)^2 + (-5 - (-3))^2}$$
$$= \sqrt{(-1)^2 + (-2)^2}$$
$$= \sqrt{5}$$

3

$$P = WX + XY + YZ + WZ$$
$$= 2\sqrt{10} + \sqrt{5} + 2\sqrt{10} + \sqrt{5}$$
$$\approx 17.1$$

The perimeter of parallelogram $WXYZ$ is approximately 17.1 units.

To determine the area of parallelogram $WXYZ$, first calculate the height, AY.

Slope of base $\overline{WX}$: $m = \frac{y_2 - y_1}{x_2 - x_1}$

$$= \frac{-3 - (-5)}{3 - (-3)}$$
$$= \frac{2}{6}$$
$$= \frac{1}{3}$$

Slope of height $\overline{AY}$: $m = -3$

Equation of base $\overline{WX}$: $(y - y_1) = m(x - x_1)$

$$(y - (-3)) = \frac{1}{3}(x - 3)$$
$$y = \frac{1}{3}x - 4$$

Equation of height $\overline{AY}$: $(y - y_1) = m(x - x_1)$

$$(y - (-5)) = -3(x - 2)$$
$$y = -3x + 1$$

Intersection of $\overline{WX}$ and $\overline{AY}$, or A: $\frac{1}{3}x - 4 = -3x + 1$

$$\frac{1}{3}x + 3x = 1 + 4$$
$$\frac{10}{3}x = 5$$
$$x = 1\frac{1}{2}$$

$$y = -3x + 1$$
$$y = -3\left(1\frac{1}{2}\right) + 1$$
$$y = -3\frac{1}{2}$$

The coordinates of point A are $\left(1\frac{1}{2}, -3\frac{1}{2}\right)$.

$$AY = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{\left(2 - 1\frac{1}{2}\right)^2 + \left(-5 - \left(-3\frac{1}{2}\right)\right)^2}$$
$$= \sqrt{\left(\frac{1}{2}\right)^2 + \left(-1\frac{1}{2}\right)^2}$$
$$= \sqrt{2.5}$$

Area of parallelogram *WXYZ*: $A = bh$

$$A = 2\sqrt{10}(\sqrt{2.5})$$
$$A = 10$$

The area of parallelogram *WXYZ* is 10 square units.

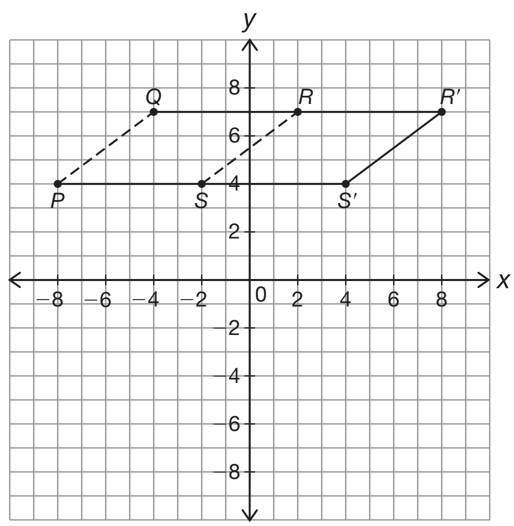
3.3 Doubling the Area of a Parallelogram

3

To double the area of a parallelogram, only the length of the bases or the height of the parallelogram needs to be doubled. If both the length of the bases and the height are doubled, the area will quadruple.

Example

Double the area of parallelogram *PQRS* by manipulating the length of the bases.



Area of *PQRS*	Area of *PQR′S′*
$A = bh$	$A = bh$
$= (6)(3)$	$= (12)(3)$
$= 18$	$= 36$

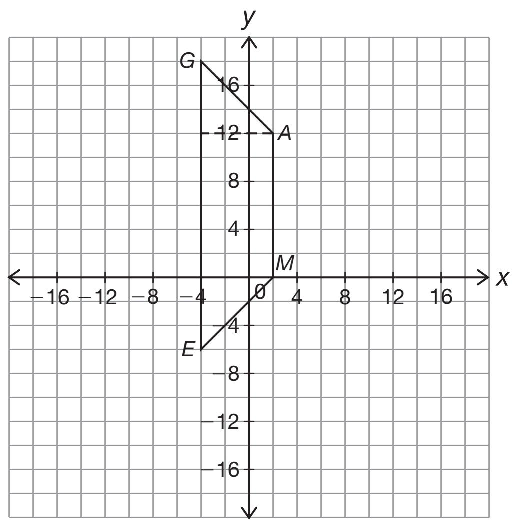
By doubling the length of the bases, the area of parallelogram *PQR′S′* is double the area of parallelogram *PQRS*.

3.4 Determining the Perimeter and Area of Trapezoids on the Coordinate Plane

A trapezoid is a quadrilateral that has exactly one pair of parallel sides. The parallel sides are known as the bases of the trapezoid, and the non-parallel sides are called the legs of the trapezoid. The area of a trapezoid can be calculated by using the formula $A = \left(\frac{b_1 + b_2}{2}\right)h$, where b_1 and b_2 are the bases of the trapezoid and h is a perpendicular segment that connects the two bases.

Example

Determine the perimeter and area of trapezoid *GAME*.



The coordinates of the vertices of trapezoid *GAME* are $G(-4, 18)$, $A(2, 12)$, $M(2, 0)$, and $E(-4, -6)$.

$$GA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(2 - (-4))^2 + (12 - 18)^2}$$
$$= \sqrt{6^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

$$ME = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{((-4) - 2)^2 + ((-6) - 0)^2}$$
$$= \sqrt{(-6)^2 + (-6)^2}$$
$$= \sqrt{72}$$
$$= 6\sqrt{2}$$

$$EG = 18 - (-6)$$
$$= 24$$

$$AM = 12 - 0$$
$$= 12$$

$$P = GA + AM + ME + EG$$
$$= 6\sqrt{2} + 12 + 6\sqrt{2} + 24$$
$$\approx 53.0$$

The perimeter of trapezoid *GAME* is approximately 53.0 units.

The height of trapezoid *GAME* is 6 units.

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$
$$= \left(\frac{24 + 12}{2}\right)(6)$$
$$= 108$$

The area of trapezoid *GAME* is 108 square units.

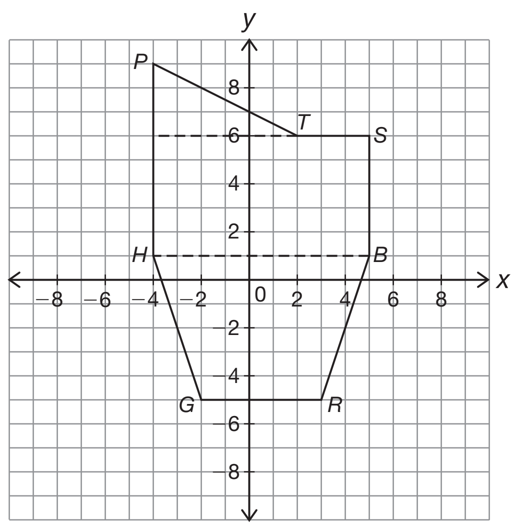
3.5 Determining the Perimeter and Area of Composite Figures on the Coordinate Plane

A composite figure is a figure that is formed by combining different shapes. The area of a composite figure can be calculated by drawing line segments on the figure to divide it into familiar shapes and determining the total area of those shapes.

3

Example

Determine the perimeter and area of the composite figure.



The coordinates of the vertices of this composite figure are *P*(−4, 9), *T*(2, 6), *S*(5, 6), *B*(5, 1), *R*(3, −5), *G*(−2, −5), and *H*(−4, 1).

$TS = 3$, $SB = 5$, $RG = 5$, $HP = 8$

$$PT = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(2 - (-4))^2 + (6 - 9)^2} = \sqrt{6^2 + (-3)^2} = \sqrt{45} = 3\sqrt{5}$$

$$BR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(3 - 5)^2 + (-5 - 1)^2} = \sqrt{(-2)^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$$

$$GH = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-4 - (-2))^2 + (1 - (-5))^2} = \sqrt{(-2)^2 + (6)^2} = \sqrt{40} = 2\sqrt{10}$$

$$P = PT + TS + SB + BR + RG + GH + HP$$
$$= 3\sqrt{5} + 3 + 5 + 2\sqrt{10} + 5 + 2\sqrt{10} + 8$$
$$\approx 40.4$$

The perimeter of the composite figure *PTSBRGH* is approximately 40.4 units.

The area of the figure is the sum of the triangle, rectangle, and trapezoid formed by the dotted lines.

Area of triangle:

$$A = \frac{1}{2}bh$$
$$= \frac{1}{2}(6)(3)$$
$$= 9$$

Area of rectangle:

$$A = bh$$
$$= 9(5)$$
$$= 45$$

Area of trapezoid:

$$A = \left(\frac{b_1 + b_2}{2}\right)h$$
$$= \left(\frac{9 + 5}{2}\right)(6)$$
$$= 42$$

The area of composite figure: $A = 9 + 45 + 42$
$= 96$

The area of the composite figure *PTSBRGH* is 96 square units.

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