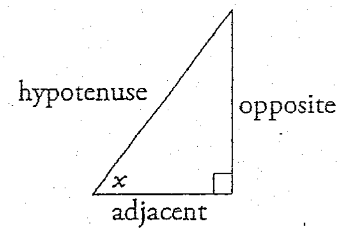
TRIGONOMETRY

SIDES OF A RIGHT TRIANGLE—RELATIONSHIPS

The easier trigonometry questions on this test involve the relationships between the sides of a right triangle. In the right triangle below, the angle x can be expressed in terms of the ratios of different sides of the triangle.



The sine of angle $x = \dfrac{\text{length of side opposite angle } x}{\text{length of hypotenuse}}$

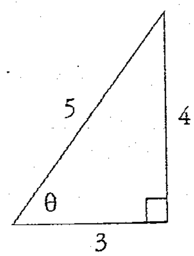
The cosine of angle $x = \dfrac{\text{length of side adjacent angle } x}{\text{length of hypotenuse}}$

The tangent of angle $x = \dfrac{\text{length of side opposite angle } x}{\text{length of side adjacent angle } x}$

There is a very handy acronym to remember all this.

SOHCAHTOA

Sine is Opposite over Hypotenuse. Cosine is Adjacent over Hypotenuse. Tangent is Opposite over Adjacent. So in the triangle on next page, the sine of angle θ [*theta*, a Greek letter] would be $\frac{4}{5}$. The cosine of angle θ would be $\frac{3}{5}$. The tangent of angle θ would be $\frac{4}{3}$.



Sine, cosine, and tangent are often abbreviated as sin, cos, and tan, respectively.

YOU'RE ALMOST DONE

There are three more relationships to memorize. They involve the reciprocals of the previous three.

$$\text{The cosecant} = \frac{1}{\text{sine}}$$

$$\text{The secant} = \frac{1}{\text{cosine}}$$

$$\text{The cotangent} = \frac{1}{\text{tangent}}$$

Let's try a few problems.

31. What is sin θ, if tan $\theta = \frac{4}{3}$?

A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{5}{4}$

D. $\frac{5}{3}$

E. $\frac{7}{3}$

Helpful Trig Identities

$\sin^2\theta + \cos^2\theta = 1$

$\frac{\sin\theta}{\cos\theta} = \tan\theta$

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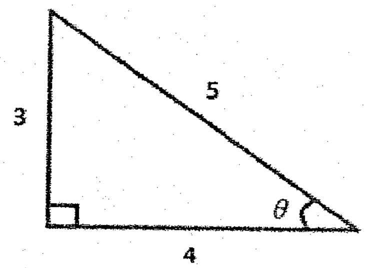
Name the sides of a triangle

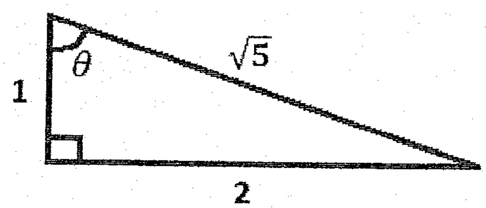
Find hypotenuse, opposite and adjacent sides from the given triangles:



Opposite side =

Adjacent side =

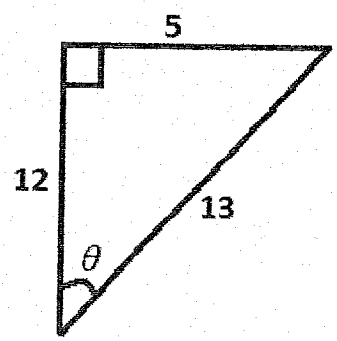
Hypotenuse =



Opposite side =

Adjacent side =

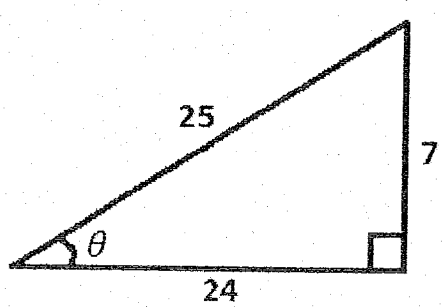
Hypotenuse =



Opposite side =

Adjacent side =

Hypotenuse =



Opposite side =

Adjacent side =

Hypotenuse =

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LESSON 13-1

Reteach

Right-Angle Trigonometry

A **trigonometric ratio** compares the lengths of two sides of a right triangle. The values of the ratios depend upon one of the acute angles of the triangle, denoted by θ.

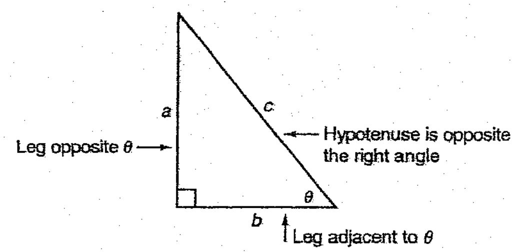
Sine is **O**pposite over **H**ypotenuse,   **C**osine is **A**djacent over **H**ypotenuse,   **T**angent is **O**pposite over **A**djacent.

Use **SOHCAHTOA** to remember the relationships between the sides of a right triangle that correspond to the trigonometric ratios sine, cosine, and tangent.

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{a}{c}$

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{b}{c}$

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{a}{b}$

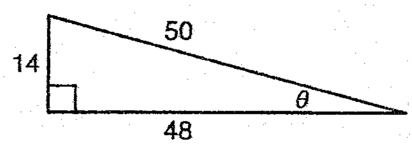


Use the definitions of each ratio and the corresponding values from a given right triangle to find the values of the trigonometric functions for θ.

$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}} = \dfrac{14}{50} = \dfrac{7}{25}$

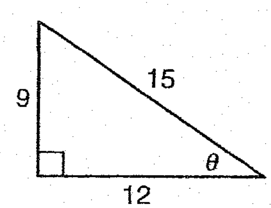
$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} = \dfrac{48}{50} = \dfrac{24}{25}$

$\tan\theta = \dfrac{\text{Opposite}}{\text{Adjacent}} = \dfrac{14}{48} = \dfrac{7}{24}$

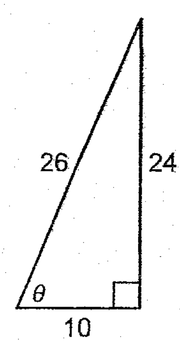


Find the value of the sine, cosine, and tangent functions for θ.

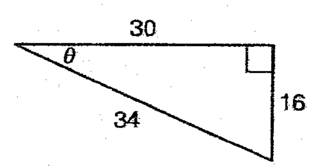
1.



2.



3.



$\sin\theta = \dfrac{\text{Opposite}}{\text{Hypotenuse}} =$ __________

$\cos\theta = \dfrac{\text{Adjacent}}{\text{Hypotenuse}} =$ __________

$\text{an}\,\theta = \dfrac{\text{Opposite}}{\text{Adjacent}} =$ __________

$\sin\theta =$ __________

$\cos\theta =$ __________

$\tan\theta =$ __________

$\sin\theta =$ __________

$\cos\theta =$ __________

$\tan\theta =$ __________

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LESSON 13-1

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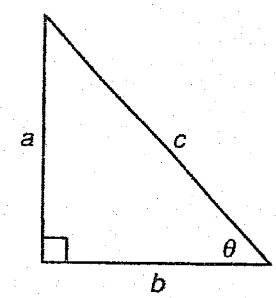
Right-Angle Trigonometry (continued)

The reciprocals of the sine, cosine, and tangent ratios are also trigonometric ratios.

The **cosecant** function (**csc** θ) is the reciprocal of the sine function.

$$\csc\theta = \frac{1}{\sin\theta} = \frac{\text{Hypotenuse}}{\text{Opposite}} = \frac{c}{a}$$



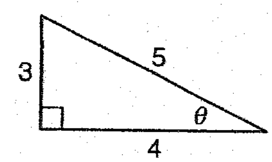
The **secant** function (**sec** θ) is the reciprocal of the cosine function.

$$\sec\theta = \frac{1}{\cos\theta} = \frac{\text{Hypotenuse}}{\text{Adjacent}} = \frac{c}{b}$$

The **cotangent** function (**cot** θ) is the reciprocal of the tangent function.

$$\cot\theta = \frac{1}{\tan\theta} = \frac{\text{Adjacent}}{\text{Opposite}} = \frac{b}{a}$$

Use the reciprocal relationship of the ratios to find the values of the reciprocal trigonometric functions.



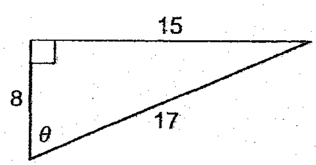
$\sin\theta = \frac{3}{5}$ $\quad\csc\theta = \frac{1}{\sin\theta} = \frac{5}{3}$

$\cos\theta = \frac{4}{5}$ $\quad\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\tan\theta = \frac{3}{4}$ $\quad\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

Find the values of the six trigonometric functions for θ.

4.

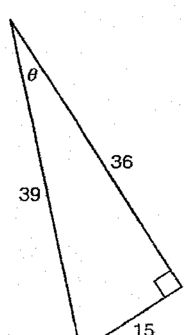


$\sin\theta =$ __________ $\quad\csc\theta = \frac{1}{\sin\theta} =$ __________

$\cos\theta =$ __________ $\quad\sec\theta = \frac{1}{\cos\theta} =$ __________

$\tan\theta =$ __________ $\quad\cot\theta = \frac{1}{\tan\theta} =$ __________

5.



__________ __________

__________ __________

__________ __________

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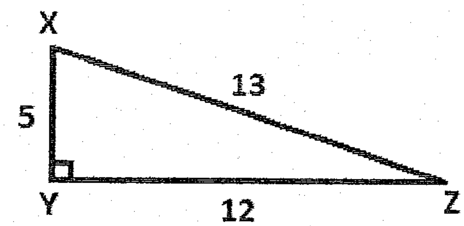
Trigonometric ratios of a triangle



Find the following trigonometric ratios given below:

sin Z = $\frac{5}{13}$	csc X =	cot Z =
cos X =	sec Z =	tan X =
cot X =	sin X =	csc Z =
tan Z =	cos Z =	sec X =

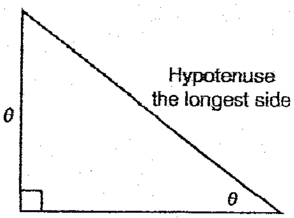
Name ______________________ Date ____________ Class ____________

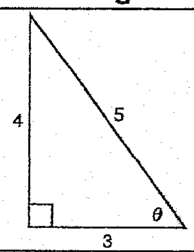
LESSON 13-1

Reading Strategy

Identify Relationships

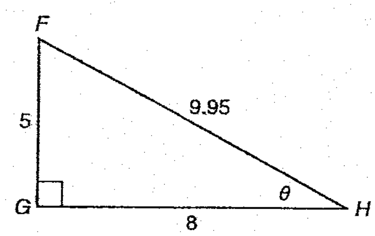
Trigonometric functions are used to describe the relationship between side length and angle measurements of right triangles. Part of finding the functions involves understanding these relationships.



Triangle	sine	cosine	tangent
	$\sin\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$ $\sin\theta = \dfrac{\text{opp.}}{\text{hyp.}} = \dfrac{4}{5}$	$\cos\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{adj.}}{\text{hyp.}} = \dfrac{3}{5}$	$\tan\theta = \dfrac{\text{opposite}}{\text{adjacent}}$ $\tan\theta = \dfrac{\text{opp.}}{\text{adj.}} = \dfrac{4}{3}$

Answer each question.

1. For triangle *FGH*



   a. What is the length of the side opposite θ? ____________

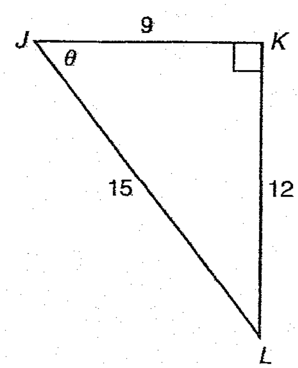
   b. What is the length of the side adjacent to θ? ____________

   c. What is the length of the hypotenuse? ____________

   d. Write a fraction to represent cosine θ. ____________

   e. Write a fraction to represent tangent θ. ____________

2. For triangle *JKL*



   a. What is the length of the side opposite θ? ____________

   b. What is the length of the side adjacent to θ? ____________

   c. What is the length of the hypotenuse? ____________

   d. Write a fraction to represent cosine θ. ____________

   e. Write a fraction to represent tangent θ. ____________

3. Compare the sine of the two acute angles in an isosceles right triangle. Explain.

____

____

Inverse Trig Functions

This solves a problem when you don't know the degree or angle.

ex: Find θ when $\cos\theta = \frac{3}{4}$

On your calculator you push [2nd] then [cos] to get $\cos^{-1}(3/4) = 41.4°$

NOTE: Make sure your calculator is in <u>degree mode</u>.

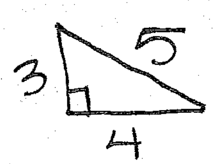
Find the following:

$\tan\theta = \frac{1}{2}$ $\quad\quad\quad$ $\sin\theta = .22$

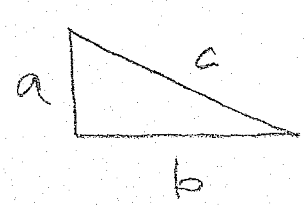
$\theta =$ ______ $\quad\quad\quad$ $\theta =$ ______

<u>3-4-5 TRIANGLES</u>

Whenever the legs of a <u>right</u> triangle are 3 and 4 the hypotenuse is always 5



Pythagorean Theorem:



$a^2 + b^2 = c^2$

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Use unit circle to find the missing ratios

Let $\cos\theta = -\frac{7}{25}, 90^0 < \theta < 180^0$

Find the value of a given trigonometric ratio using unit circles:

$\sin\theta =$

$\tan\theta =$

$\sec\theta =$

$\csc\theta =$

$\cot\theta =$

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LESSON 13-3

Reteach

The Unit Circle

Radians are a real number measure of rotation.
To convert between radians and degrees, use the following identity.

π radians = 180°

To convert from degrees to radians, solve the identity for 1 degree.	To convert from radians to degrees, solve the identity for 1 radian.
$1 \text{ degree} = \frac{\pi \text{ radians}}{180°}$	$1 \text{ radian} = \frac{180°}{\pi \text{ radians}}$

Convert 60° to radians.

$60° = \cancel{60°}^{1}\left(\frac{\pi \text{ radians}}{\cancel{180°}_{3}}\right) = \frac{\pi}{3} \text{ radians}$

Use dimensional analysis to help. Notice that the degrees cancel so the remaining unit is radians.

Convert $\frac{5\pi}{4}$ radians to degrees.

$\frac{5\pi}{4} \text{ radians} = \left(\frac{5\cancel{\pi}}{\cancel{4}} \cancel{\text{radians}}\right)\left(\frac{^{45}\cancel{180°}}{\cancel{\pi} \cancel{\text{radians}}}\right) = 225°$

The radians cancel so the remaining unit is degrees.

Convert each measure from degrees to radians.

1. −45°

   $-45° = -45°\left(\frac{\pi \text{ radians}}{180°}\right)$

   ____

2. 150°

   $150° = 150°\left(\frac{\pi \text{ radians}}{180°}\right)$

   ____

3. 210°

   ____

4. −120°

   ____

Convert each measure from radians to degrees.

5. $\frac{4\pi}{3}$ radians

   $\frac{4\pi}{3} \text{ radians} = \left(\frac{4\pi}{3} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right)$

   ____

6. $-\frac{3\pi}{2}$ radians

   $-\frac{3\pi}{2} \text{ radians} = \left(-\frac{3\pi}{2} \text{ radians}\right)\left(\frac{180°}{\pi \text{ radians}}\right)$

   ____

7. $\frac{\pi}{6}$ radians

   ____

8. $\frac{5\pi}{3}$ radians

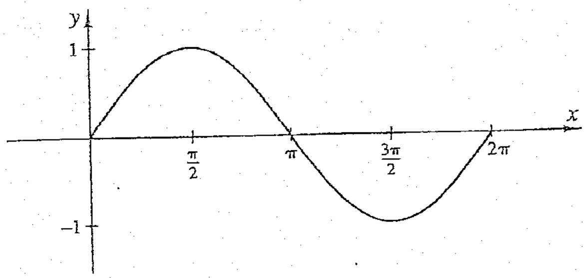
   ____

13-4

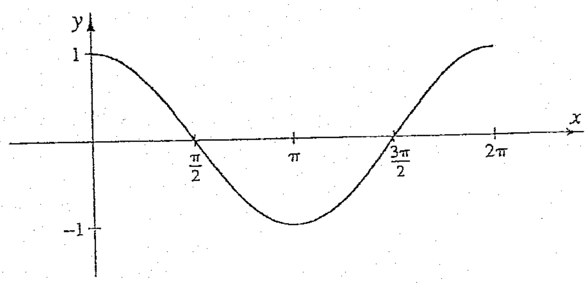
Graphs of Trigonometric Functions

Trigonometric functions repeat themselves. The **period** of a trigonometric function is the distance required to show one full cycle. You should be able to recognize the graphs of trigonometric functions. Those shown below are for one period of each function.

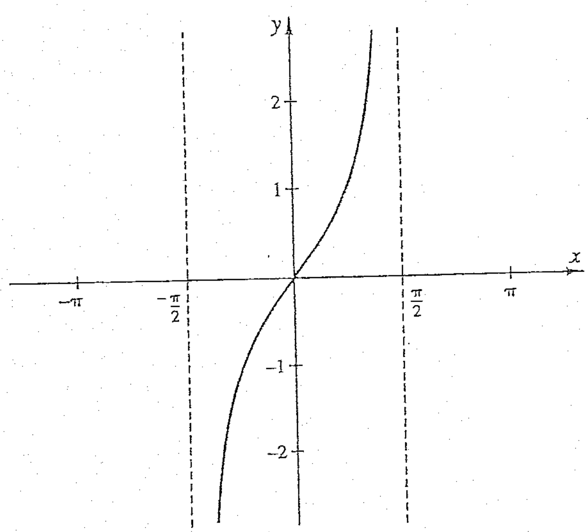
$y = \sin x$



$y = \cos x$



$y = \tan x$



$y = a\sin(bx)$ or $y = a\cos(bx)$

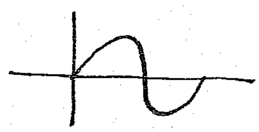
Amplitude is a

$$\frac{\text{Max} - \text{Min}}{2} = a$$

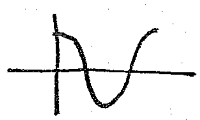
Period is $\frac{2\pi}{b}$

↳ How long it takes for the graph to make one full cycle

Sin:



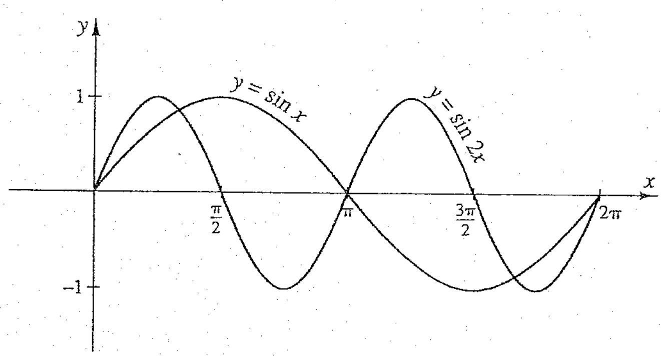
Cos:



The period of a trigonometric graph changes depending on the coefficient of x.

Example

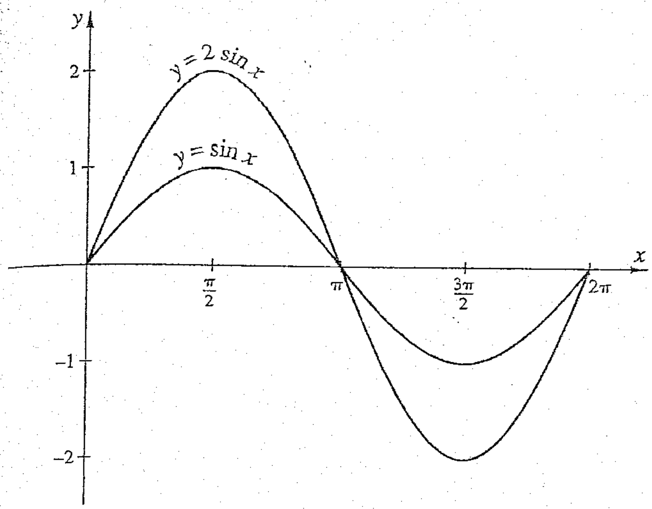
The function $y = \sin 2x$ has a period that is *half* as long as the period of $y = \sin x$.



The **amplitude** of a sine or cosine curve is half the distance between the smallest and largest y values for the function. The amplitudes of the sine and cosine graphs change depending on the coefficient of sine or cosine. The amplitude for tangent, cotangent, secant, and cosecant is undefined.

Example

The function $y = 2\sin x$ has an amplitude that is twice the amplitude of $y = \sin x$.



MODEL ACT PROBLEMS

1. What is the period of the graph $y = 4\tan 2x$?

   A. 4π

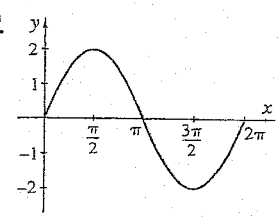
   B. 4

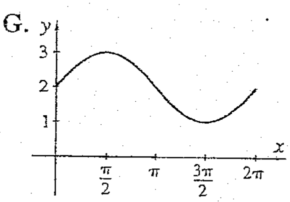
   C. π

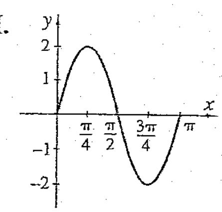
   D. 2

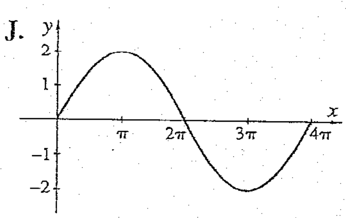
   E. $\frac{\pi}{2}$

2. Which graph below shows one period of $y = 2\sin x$?

F. 

G. 

H. 

J. 

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